COMBINED MATCHED FILTER AND POLYNOMIAL-BASED INTERPOLATOR FOR SYMBOL SYNCHRONIZATION IN DIGITAL RECEIVERS

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ABSTRACT

In digital receivers where the sampling is not synchronized to the received data, the symbol synchronization is usually done using interpolation. From the implementation point of view, the polynomial-based interpolation is preferred because it can be efficiently implemented using the Farrow structure.

This paper shows how the receiver complexity can be reduced by combining the matched filter and the polynomial-based interpolator. An algorithm is described for optimizing the combined matched filter/interpolator system. It is shown, through the simulations, that there is only a negligible increase in the mean-square error at the output of the combined matched filter/interpolator compared to the case where separate filters are used.

1. INTRODUCTION

In digital receivers, the trend is to move the analog-to-digital converter closer to the antenna in order to enable us to reduce the number of analog components and to perform most of the operations digitally. One key point is to use a receiver architecture where the sampling is not synchronized to the received data symbols.

In this kind of receivers, the incoming signal is sampled by using a fixed sampling clock followed by a matched filter, which is usually square-root raised cosine filter. After the matched filter, the symbol synchronization is performed by using interpolator [1],[2].

The receiver can be simplified by integrating the matched filter and interpolator. For simplicity, the combined matched filter/interpolator is referred to as a *combined filter*. Several synthesis methods for combined filters are introduced in [3]-[6]. In [3], the implementation structure is a polyphase filter or FIR filter where the coefficients are calculated on-line. The use of B-spline functions for design combined filters is presented in [4].

The idea of this paper is to combine the matched filter to the polynomial-based interpolator. The optimization of the combined filter is considered and it is shown that this combination reduces the complexity of the receiver without increasing the mean-square error (MSE) at the output of the interpolator. Because the combined filter is a poly-nomial-based filter, it can be efficiently implemented using the Farrow structure [7].

2. COMBINED MATCHED FILTER AND INTERPOLATOR

Figure 1 shows a simplified digital receiver with nonsynchronized sampling. The sampling of the received signal $r(t)$ is performed by a nonsynchronized sampling clock followed by a matched filter with a impulse response $h_R(n)$. Because the matched filter is after the sampling, the sampling rate $F_s=1/T_s$ has to be fixed. In this paper, it is assumed that the sampling rate $1/T_s$ is twice the symbol rate $1/T$.

Usually the transmit filter with an impulse response $h_T(n)$ and the receive filter with an impulse response $h_R(n)$ together form a Nyquist filter having the zero inter-symbol interference (ISI) and the receive filter is matched to the transmit filter, that is, their impulse responses $h_R(n) = h_T(N-1-n)$, where N is the length of the filters.

In this paper, it is assumed that the transmit and receive filters are square-root raised cosine filters with a roll-off factor α. In the ideal case, the square-root raised cosine filters are of infinite length. However, because of a finite duration of the impulse response for filters in the practical implementation, there is some ISI.

The timing adjustment is done after the sampling and matched filter by utilizing an interpolator having the underlying continuous-time impulse response $h(t)$. The output samples $y(k)$ of the interpolator (one sample per symbol interval) are located at the estimated symbol time instants $t = kT$, which are expressible as

$$kT = (n + \mu)T_s, \tag{1}$$

where $\mu \in [0,1)$ is the *fractional interval* (or timing error estimate), T_s is the sampling interval, and T is the symbol interval, whereas n is the largest integer for which $nT_s \leq kT$.

In Fig. 2, the matched filter is combined with the interpolator to reduce the receiver complexity. This combined filter with the impulse response $h_C(t)$ should have approximately a zero ISI together with the transmit
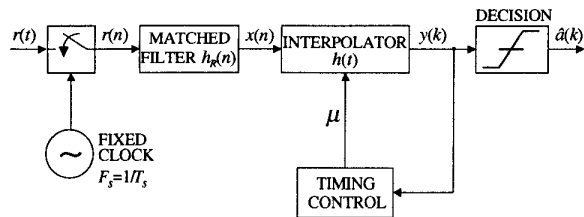
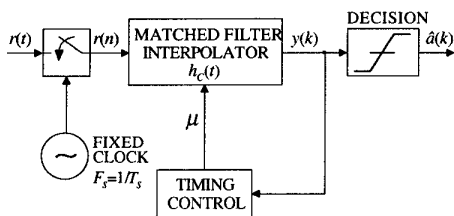Fig. 1. Digital receiver with a nonsynchronized sampling.

Fig. 2. Digital receiver with combined matched filter and interpolator.

Fig. 3. The Farrow structure.

Fig. 4. Hybrid analog/digital model for the interpolation filter.

filter, it should be matched to the transmit filter, and it should have a good interpolation performance.

The combination of the matched filter and interpolator is reasonable only if it has the same or lower complexity than the separate filters together with the same performance in terms of the MSE or the bit error rate (BER).

3. IMPLEMENTATION OF THE INTERPOLATOR

The interpolator in Fig. 1 and the combined filter in Fig. 2 can be efficiently implemented using the Farrow structure [7] if they are polynomial-based filters, that is, $h(t)$ and $h_C(t)$ are piecewise polynomials.

In the general case (see Fig. 3), the Farrow structure consist of $L+1$ FIR filter branches. The transfer functions of these FIR filters are given by

$$C_l(z) = \sum_{n=0}^{N-1} c_l(n) z^{-n} \quad \text{for} \quad l = 0, 1, \cdots, L, \tag{2}$$

where L is the degree of the interpolation and NT_s is the length of the impulse response $h(t)$. In the modified Farrow structure [8], μ is replaced by $2\mu-1$ and the $L+1$ FIR filters are linear phase Type II and IV filters. By exploiting the coefficient symmetry the number of multiplications in the FIR filters is $(L+1)N/2$ per output sample.

The analysis and synthesis of the Farrow structure is difficult because its impulse response is a function of μ. Therefore, the Farrow structure is usually modeled by a simple hybrid analog/digital interpolation filter (see Fig. 4), where the analog signal $y(t)$ is reconstructed using a D/A-converter and analog anti-imaging filter having a piecewise polynomial impulse response $h(t)$. The interpo-
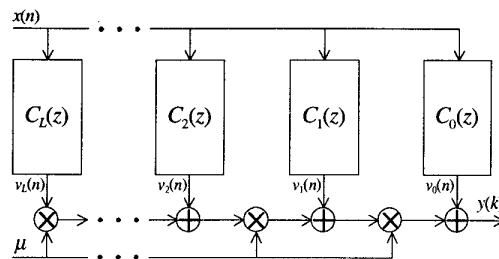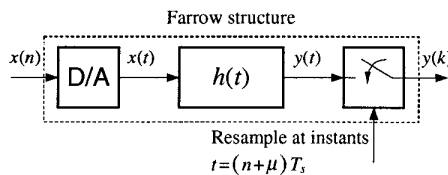
lated sample values $y(k)$ are then obtained by resampling this analog signal at time instants $t = (n+\mu)T_s$.

4. FILTER OPTIMIZATION

When optimizing the combined filter having a piecewise polynomial impulse response $h_C(t)$, it is assumed that the transmit and receive filters are square-root raised cosine filters with a roll-off factor α and the sampling rate is twice the symbol rate. However, the optimization procedure can be easily extended for other pulse shapes and sampling rate factors as well.

As was mentioned earlier, the combined filter should have three properties. First, the convolution between the received sampled pulse shape $r(n) = h_T((n+\mu)T_s)$ and the impulse response of the combined filter $h_C(n) = h_C((n+\mu)T_s)$ should have the zero ISI for all the values of the timing error μ. Second, the combined filter should maximize the signal-to-noise ratio (SNR) at the symbol time instants. This means that the impulse response of the combined filter $h_C(t)$ is matched to the transmit filter $h_T(t)$, i.e., $h_C(t) = h_T(NT_s-t)$. Finally, the combined filter should have a good interpolation performance, which means that it has a good attenuation at the image frequencies.

Based on these three properties, the coefficients of the combined filter $c_l(n)$ are optimized in such a way that the ISI and the error caused by a non-ideal interpolation are minimized and, at the same time, $h_C(t)$ is matched to $h_T(t)$. The optimization can be done using the least-mean-square criterion and, thus, the overall optimization problem can be stated as follows:

Least-Mean-Square Optimization Problem: Find the $(L+1)N/2$ unknown coefficients $c_i(n)$ for the combined filter to minimize the following error function:

$$E = W_{matched}E_{matched} + W_{ISI}E_{ISI} + W_{freq}E_{freq}, \qquad (3)$$

where $E_{matched}$ is the squared error between the impulse response of the matched filter $h_T(NT_s-t)$ and that of the combined filter $h_C(t)$, E_{ISI} is the squared error at the decision points caused by the ISI, and E_{freq} is the squared and weighted error between the frequency response of the combined filter $H_C(f)$ and the desired response $D(f)$. $W_{matched}$, W_{ISI}, and W_{freq} are the positive coefficients which can be used to give different weights for the squared errors $E_{matched}$, E_{ISI}, and E_{freq}, respectively.

The three squared error components in Eq. (3) can be given by

$$E_{matched} = \int_0^{NT_s} \left(h_C(t) - h_T(NT_s - t)\right)^2 dt, \qquad (4)$$

$$E_{ISI} = \int_0^1 \sum_{n=0}^{N-1} \left(\sum_{k=0}^{N-1} h_T\left((k+\mu)T_s\right)h_C\left((2n-k+\mu)T_s\right) - \delta(n-\tfrac{N}{2})\right)^2 d\mu, \qquad (5)$$

and

$$E_{freq} = \int_{f \in X} \left(W(f)(H_C(f) - D(f))\right)^2 df, \qquad (6)$$

where NT_s is the length of the impulse response of the combined filter, $\delta(n)$ is the unit impulse, X is the approximation region consisting of passband and stopband, and $W(f)$ is a positive weighting function.

The impulse and frequency responses of the combined filter as well as the desired response are given by

$$h_C(t) = \sum_{n=0}^{N/2-1} \sum_{l=0}^{L} c_l(n)g(n,l,T_s,t), \qquad (7)$$

$$H_C(f) = \sum_{n=0}^{N/2-1} \sum_{l=0}^{L} c_l(n)G(n,l,T_s,f), \qquad (8)$$

and

$$D(f) = \begin{cases} 1 & \text{for } 0 \le f \le (1+\alpha)F_s/4 \\ 0 & \text{for } F_s - (1+\alpha)F_s/4 \le f \le \infty, \end{cases} \qquad (9)$$

where $g(n,l,T_s,t)$ are the so-called basis functions having the frequency responses of $G(n,l,T_s,f)$ (see [8]), L is the degree of interpolation, and α is the roll-off factor of the matched filter.

5. DESIGN EXAMPLES AND SIMULATIONS

This section compares, by means of simulations, the MSE of the combined filter to that of the case with separate filters. In the simulation model, a binary PAM signal with a square-root raised cosine pulse shape is received. The roll-off factor is $\alpha = 0.4$ and the channel is assumed to be ideal.

The received signal is sampled using a nonsynchronized sampling and two samples per symbol. After the sampling, the timing adjustment is done using the combined filter. The MSE is calculated between the ideal symbol values and the sample values occurring at the output of the combined filter (one sample per symbol). The timing error estimate is assumed to be ideal. The MSE is also calculated in the case of separate matched filter and an interpolator.

The coefficients of the separate interpolator used in the simulations are optimized in the least-mean-square sense and the combined filter is designed using the above-mentioned procedure.

The length of the combined filter is $12T_s$ ($N = 12$) and the degree of the interpolation is $L = 2$, whereas, for the separate interpolator, $N = 10$ and $L = 3$. Consequently, the total number of coefficients in the Farrow structure is 36 and 40 for the combined and separate cases, respectively. The length of the matched filter is 21 and, therefore, the total number of coefficients in the separate case is 61.

The MSE as a function of μ in the separate and combined cases are shown in Fig. 5. As can be seen, the MSE is almost the same for the both cases. This means that the receiver complexity can be reduced by combining the matched filter and interpolator without increasing the MSE. In this example, the number of coefficients is reduced from 61 to 36.

The frequency responses of the combined filter and separate interpolator are shown in Figs. 6 and 7, respectively. The corresponding impulse responses are show in Fig. 8.
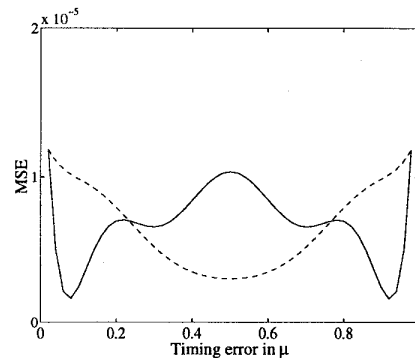

Fig. 5. MSE at the output of the interpolator for the combined filter (solid line) and separate filters (dashed line).

6. CONCLUSIONS

The optimization of the combined matched filter and polynomial-based interpolator has been presented. In the optimization, the error function to be minimized consists of three components which ensure that the required three properties for the combined filter are achieved.

It has been shown that the complexity of the receiver can be considerably reduced by combining the matched filter and the interpolator without increasing the MSE.

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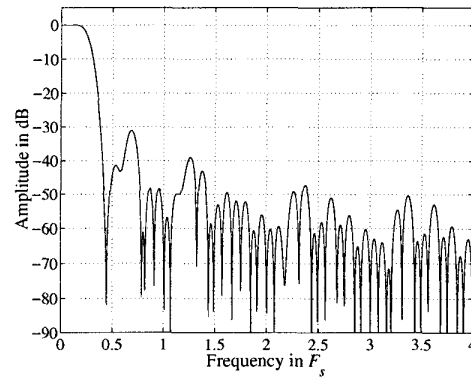
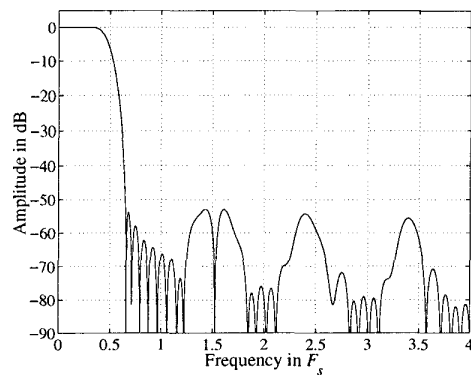Fig. 6. Frequency response $H_C(f)$ for the combined filter.

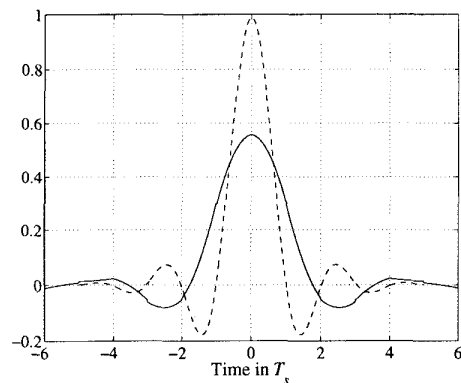Fig. 7. Frequency response $H(f)$ for the separate interpolator.

Fig. 8. The impulse response for the combined filter (solid line) and separate interpolator (dashed line).