Effect of Noisy Phase Reference on Coherent Detection of Band-limited Offset-QPSK Signals

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Abstract—**In this paper, the performance of band-limited offset quaternary phase shift keying (offset-QPSK, OQPSK) systems in the presence of noisy phase reference and additive Gaussian noise is analyzed. Expressions for error probability calculation are obtained. A new search method is proposed for computing the detection loss and the required SNR of the phase reference. It is shown that when the rolloff factor for the Nyquist filtering is large, OQPSK surpasses QPSK, but when the rolloff factor is small, QPSK surpasses OQPSK.**

I. Introduction

The purpose of this paper is to investigate the effect of noisy phase reference on the performance of band-limited offset quaternary phase shift keying (offset-QPSK or OQPSK) data transmission systems. In conventional quaternary phase shift keying (QPSK) modulation, the two binary data streams are time coincident, whereas the bit alignments are staggered for OQPSK modulation. Rhodes [1] has shown that OQPSK requires less signal-to-noise ratio of phase reference than QPSK for the same error probability performance, but his analysis is only for "infinite" bandwidth systems. Gitlin and Ho [2] analyze the performance of band-limited OQPSK systems in the presence of phase jitter, but the criterion used is the mean square error. Using this criterion, they conclude that OQPSK surpasses QPSK for any rolloff factor for the Nyquist filtering. Palmer *et al.* [3] discuss the problem of synchronization for band-limited QPSK and OQPSK, but only computer simulation results are given. In this paper, we analyze the error probability performance of band-limited OQPSK systems in the presence of noisy phase reference. It is shown that for large rolloff factor, OQPSK surpasses QPSK, but for small rolloff factor, the reverse is true.

In Section II, the system model is described and expressions for error probability calculation are obtained. In Section III, methods of computation are discussed and various performance curves are given. Section IV is for the conclusions.

II. Analysis

The QPSK system we analyzed is shown in Fig. 1. The channel and all filters are assumed to be linear and time invariant. For OQPSK, a $(T/2)$-delayer should be added between the receive filter and the sampler in the in-phase channel in order to recover the bit alignments where $1/T$ is the symbol rate. For both QPSK and OQPSK modulation, the input of the transmit filter can be written as

$$W(t) = \sum_{k=-\infty}^{\infty} a_k \operatorname{rec}[(t-kT)/T] \cos \omega_c t + \sum_{l=-\infty}^{\infty} b_l \operatorname{rect}[(t-lT-DT)/T] \sin \omega_c t \quad (1)$$

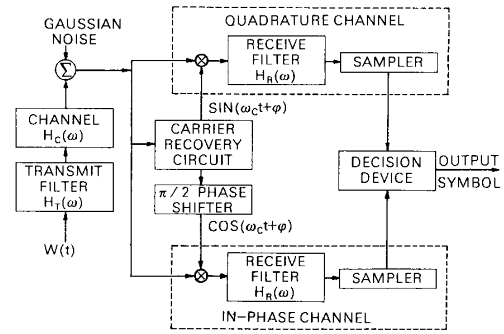

Fig. 1. QPSK receiver for band-limited system.

where $D = 1/2$ for OQPSK, $D = 0$ for QPSK, $\operatorname{rect}(x) = 1$ for $|x| < 1/2$, $\operatorname{rect}(x) = 0$, otherwise. $\omega_c = 2\pi f_c$, f_c is the carrier frequency. a_k and b_l are assumed to be independent binary random variables which take the values 1 or -1 with equal probability. The received signal $y(t)$ at the output of the receive filter in the in-phase channel can be written as

$$y(t) = \{[W(t) * h_T(t) * h_c(t) + n(t)] \cos(\omega_c t + \varphi)\} * h_R(t) \quad (2)$$

where $h_T(t)$, $h_C(t)$, and $h_R(t)$ are the impulse responses of the transmit filter $H_T(\omega)$, the channel $H_C(\omega)$ and the receive filter $H_R(\omega)$, respectively. φ is the phase error relative to the modulator reference phase. $n(t)$ is the Gaussian noise with autocorrelation given by $R(\tau) = (N_0/2)\delta(\tau)$ where $\delta(\cdot)$ is the Dirac delta-function.

The transmit and receive filter are designed such that the overall frequency characteristic $H(\omega)$ has the well-known form of the raised-cosine Nyquist filtering.

$$H(\omega) = \begin{cases} T, & \\ \quad |\omega T| \le \pi(1-\beta) & \\ T \cdot \cos^1 [(|\omega T| - \pi(1-\beta)]/(4\beta), & \\ \quad \pi(1-\beta) < |\omega T| < \pi(1+\beta) & \\ 0, & \\ \quad |\omega T| \ge \pi(1+\beta) & \end{cases} \quad (3)$$

where $0 < \beta \le 1$ is the rolloff factor for the Nyquist filtering, $H(\omega) = \operatorname{RECT}(\omega) H_{\mathrm{TCL}}(\omega) H_R(\omega)$, $\operatorname{RECT}(\omega)$ is the Fourier transform of $\operatorname{rect}(t/T)$ and $H_{\mathrm{TCL}}(\omega)$ is the equivalent low-pass frequency characteristic of $H_T(\omega) \cdot H_C(\omega)$. Hypothetically, $H_R(\omega)$ is directly proportional to the square root of $H(\omega)$. The impulse resposne $h(t)$ which is the Fourier transform of $H(\omega)$ is shown in Fig. 2. We assume that the symbol synchronization is perfect. $y(t)$ is sampled at $t = nT$ where decisions are made by detecting a "1" if $y(nT)$ lies in the range $0 \le y(nT) < \infty$ and a "-1" if $-\infty < y(nT) < 0$. The error probability in the in-phase channel conditioned on phase

Paper approved by the Editor for Data Communications and Modulation of the IEEE Communications Society. Manuscript received February 23, 1988; revised March 15, 1989. This paper was presented in part at the International Conference on Communication Technology, Nanjing, China, November 9-11, 1987.

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IEEE Log Number 8933279.

0090-6778/90/0200-0156$01.00

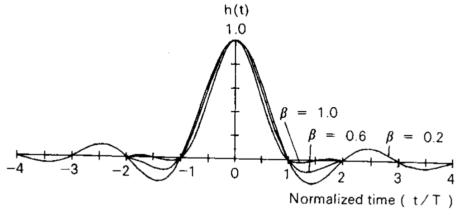

Fig. 2. Raised-cosine pulse responses for various values of β.

reference error φ can be shown as

$$P(\varphi) = \frac{1}{2}\left\{Q\left[\sqrt{\frac{2E_b}{N_0}}(\cos\varphi - \sin\varphi)\right] + Q\left[\sqrt{\frac{2E_b}{N_0}}(\cos\varphi + \sin\varphi)\right]\right\}, \quad \text{for QPSK}, \tag{4a}$$

$$P(\varphi) = \left\langle Q\left\{\sqrt{\frac{2E_b}{N_0}}\left[\cos\varphi - \sin\varphi\sum_{l=-\infty}^{\infty} b_l h \cdot \left(-lT - \frac{T}{2}\right)\right]\right\}\right\rangle_{\cdots b_{-1}, b_0, b_1, \cdots} \quad \text{for OQPSK} \tag{4b}$$

where E_b/N_0 is the received ratio of signal energy per bit-to-noise power density, $\langle\cdot\rangle_z$ denotes the average over Z, $Q(x)$ is the Gaussian integral

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp(-U^2/2)\,dU.$$

The error probability in the quadrature channel can easily be shown to be identical to that in (4). Therefore, (4) is the expression for the bit error probability conditioned on phase reference error Φ.

In (4a), $P(\varphi)$ does not depend on b_l ($l \neq 0$, $l = \cdots, -2, -1, 1, 2, \cdots$) in the detection of a_0 for reason as follows. With the assumption of Nyquist filtering and perfect symbol synchronization, there is no intersymbol interference (ISI) for QPSK at the detection sampling instants. When $\varphi \neq 0$, the only interference in the detection of a_0 is from b_0 and this leads the $-\sin\varphi$ and $+\sin\varphi$ terms in (4a). Then, the error performance of QPSK is not influenced by the bandwidth limitation of Nyquist filtering and is therefore independent of the folloff factor β. From (4a) we also see that the error probability of band-limited QPSK is identical to that of infinite bandwidth QPSK analyzed in [1].

For OQPSK, because of the timing offset of $T/2$ for the binary components, the correct sampling points for bit decisions on one binary component does not occur at ISI nulls for pulses of the other binary component. Therefore, in (4b), $P(\varphi)$ depends on b_l ($l = \cdots, -2, -1, 0, 1, 2, \cdots$). The magnitude of $h(lT - T/2)$ increases as the rolloff factor β is decreased, so the detection performance of OQPSK is better as β is larger which will be shown in Fig. 3. When $\beta = 1$, from Fig. 2 it can be seen that $h(-lT - T/2) = 0$ except for $l = -1, 0$ and $h(\pm T/2) = 1/2$. The detection of a_0 is only influenced by b_{-1} and b_0. If the polarity of b_{-1} is opposite to that of b_0, the interference of b_{-1} is cancelled by the interference of b_0 and hence the detection performance is identical to that for BPSK. If b_{-1}, b_0 have the same polarity, the interference is $[h(T/2) + h(-T/2)]\sin\varphi = \sin\varphi$ or $-\sin\varphi$ and the detection performance is identical to that for QPSK. Thus, when $\beta = 1$, (4b) can be simplified as

$$P(\varphi) = \frac{1}{2}Q[\sqrt{2E_b/N_0}\cos\varphi] + \frac{1}{4}Q[\sqrt{2E_b/N_0}(\cos\varphi - \sin\varphi)] + \frac{1}{4}Q[\sqrt{2E_b/N_0}(\cos\varphi + \sin\varphi)], \quad \text{for OQPSK}, \beta = 1 \tag{4c}$$

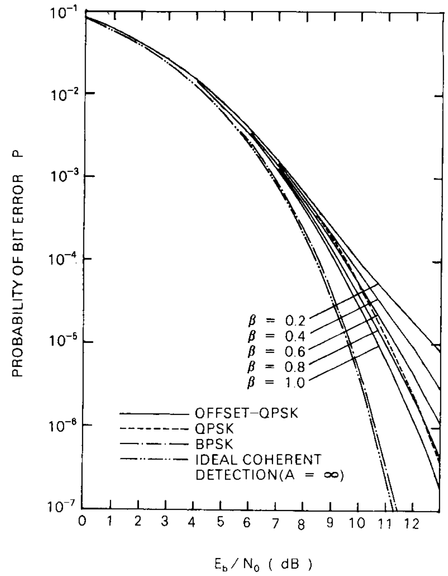

Fig. 3. Detection performance for band-limited systems when SNR of phase reference $A = 10 \cdot \log(\alpha) = 19$ dB.

which is identical to that for infinite bandwidth OQPSK analyzed in [1].

The error probability of OQPSK given by (4b) is too time-consuming to compute. This problem can be solved by Gram-Carlier series expansion method [4]. According to [4], the error probability of OQPSK can be written as follows:

$$P(\varphi) = Q(\sqrt{2E_b/N_0}\cos\varphi) - (1/\sqrt{2\pi})\exp[-(E_b/N_0)\cos^2\varphi] \cdot \left[\sum_{k=1}^{\infty}(2E_b/N_0)^k \cdot H_{2k-1}(\sqrt{2E_b/N_0}\cos\varphi)\frac{M_{2k}}{(2k)!}\right] \tag{5}$$

where $H_{2k-1}(x)$ is the Hermite polynomial, M_{2k} is the $2k$th moment of the random variable $[\sin\varphi\sum_{l=-\infty}^{\infty} b_l h(lT - T/2)]$. M_{2k} can be computed according to a recurrence relation [4] given by

$$M_{2k} = \sum_{l=1}^{k}\begin{bmatrix}2k-1\\2i-1\end{bmatrix}(-1)^{i+1}M_{2(k-i)}f^{(2i-1)}(0) \tag{6a}$$

$$f^{(2i-1)}(0) = \frac{4^i(4^i-1)}{2i}B_l\sum_{l=-\infty}^{\infty}[\sin\varphi \cdot h(lT - T/2)]^{2i} \tag{6b}$$

where B_i is Bernoulli number, $M_0 = 1$.

In the consumption of error probability, the carrier tracking phase error is treated as a constant over each symbol interval, which is virtually true for synchronizer bandwidths that do not exceed one tenth of the digital signaling rate. The average probability of error is then found by integrating (4a), (4c), or (5) over the probability density function (pdf) $f_\Phi(\varphi)$ of the random phase error Φ.

$$P = \int_{-\pi}^{\pi} P(\varphi)f_\Phi(\varphi)\,d\varphi \tag{7}$$

III. Computation

In the computation of error probability, we assume that the pdf of the phase tracking error is [1].

$$f_\Phi(\varphi) = \frac{\exp(\alpha \cdot \cos\varphi)}{2\pi I_0(\alpha)}, \quad -\pi < \varphi < \pi. \tag{8}$$

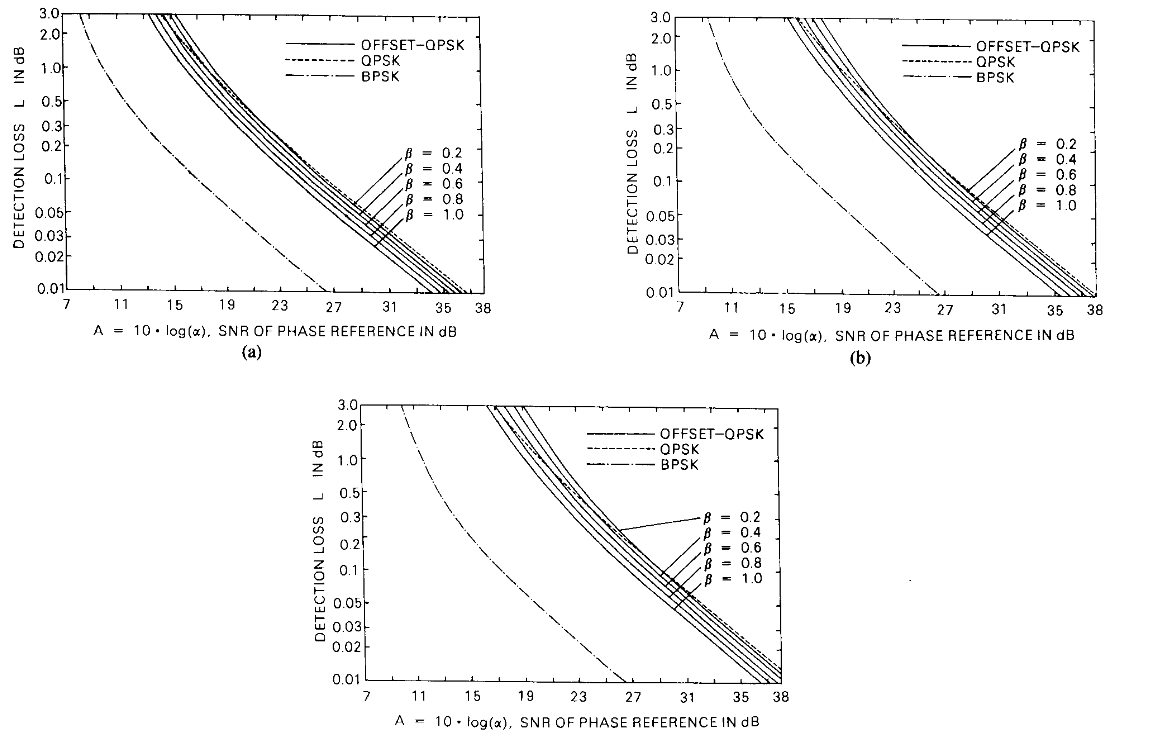

Fig. 4. Detection loss in band-limited systems as a function of SNR of phase reference. (a) Evaluated at $P_0 = 10^{-3}$. (b) Evaluated at $P_0 = 10^{-4}$. (c) Evaluated at $P_0 = 10^{-5}$.

where α is the SNR or the phase reference, which is approximately equal to the inverse of the variance of Φ. I_0 refers to a zero-order modified Bessel function of the first kind.

Fig. 3 shows the calculated error probability P versus E_b/N_0 and parametric in the rolloff factor β, when the SNR of phase reference is 19 dB ($A = 10\log(\alpha) = 19$ dB). The result for BPSK is also shown for comparison. The performance of Nyquist filtered BPSK is independent of β and identical to that of infinite bandwidth BPSK for the similar reason described in Section II for QPSK. So it is seen that for BPSK and QPSK, P does not vary with β. For OQPSK, the larger the rolloff factor is, the better is the detection performance. For large rolloff factor, OQPSK surpasses QPSK, but for small rolloff factor, the reverse is true. In the calculation, we use 10 terms in (5) and 41 terms in (6b) which are found to be appropriate and discussed in [5]. A recurrence relation of $M_{2k}/k!$ is actually adopted instead of M_{2k} given by (6a) to limit the calculated numbers not too large to overflow from the computer (VAX-11/780) capacity.

The loss in detection efficiency that is associated with a noisy phase reference is defined as the required increase in E_b/N_0 relative to ideal coherent detection for maintaining a given error probability P_0. For band-limited BPSK and QPSK, the loss in detection efficiency can be expressed by a function such as $L = \boldsymbol{F}(\beta, P_0)$ like those for infinite bandwidth BPSK and QPSK given in [1]. Unfortunately, the function $\boldsymbol{F}$ cannot be written explicitly for band-limited OQPSK because of the complexity of (5). We use a kind of "searching" method to calculate the loss in detection efficiency L as follows. Suppose that a function $G(\Delta)$ is defined as

$$G(\Delta) = \mathrm{MAX}(P_0/P, P/P_0) \tag{9}$$

where $P_0 = \boldsymbol{P}_0(E_b/N_0)$, $P = \boldsymbol{P}\{[(E_b/N_0) + \Delta], \beta, \alpha\}$. G can be viewed as a function of Δ when the values of E_b/N_0, β and α are fixed. The minimum of G is 1.0 and the value of Δ which enables G equal 1.0 is L, the loss in detection efficiency. G increases monotonically if Δ deviates from L, i.e., $G(\Delta)$ is a unimodal function of the continuous variable Δ defined on a closed interval, for instance, [0, 6 dB]. A computer program is designed based upon a method of optimization, Golden Section search [6], for searching the value of Δ which enables G approximately equal to 1.0. We denote this Δ be $\hat{L}$. The value of $G(\hat{L})$ is an indication of the accuracy of the search. Detection losses for band-limited BPSK and QPSK systems are also calculated by this searching program for comparison. Fig. 4 shows the detection losses versus SNR of the phase reference. The maximum among the values of $G(\hat{L})$ equals 1.00037, which indicate the high accuracy of the search.

In (9), G can also be viewed as a function of α when the values of E_b/N_0, β and Δ are fixed, i.e., $G(\alpha) = \mathrm{MAX}(P_0/P, P/P_0)$. The value of α which enables $G = 1.0$ is the required value of SNR of phase reference at a certain fixed loss $\Delta = L$ in detection. Fig. 5 shows the required SNR of the phase reference versus the error probability when the detection loss is specified.

IV. Conclusions

Imperfect carrier synchronization causes a performance loss for coherent band-limited PSK system. The bit error probability can be calculated by (7) and (4a) for QPSK systems and by (7) and (5) or (4c) for OQPSK systems. For band-limited BPSK or QPSK signaling, the performance loss is independent of the rolloff factor and is identical to that for infinite bandwidth BPSK or QPSK. However, for bandlimited OQPSK signaling, the performance loss increases as the rolloff factor β is decreased. When β equals 1, the performance loss for band-limited OQPSK is identical to that for infinite bandwidth OQPSK.

It is shown that when β is large, OQPSK surpasses QPSK and when β is small, QPSK surpasses OQPSK. The turning value of β for which the performance of OQPSK equals that of QPSK depends on the specified error probability and the SNR of the phase reference.

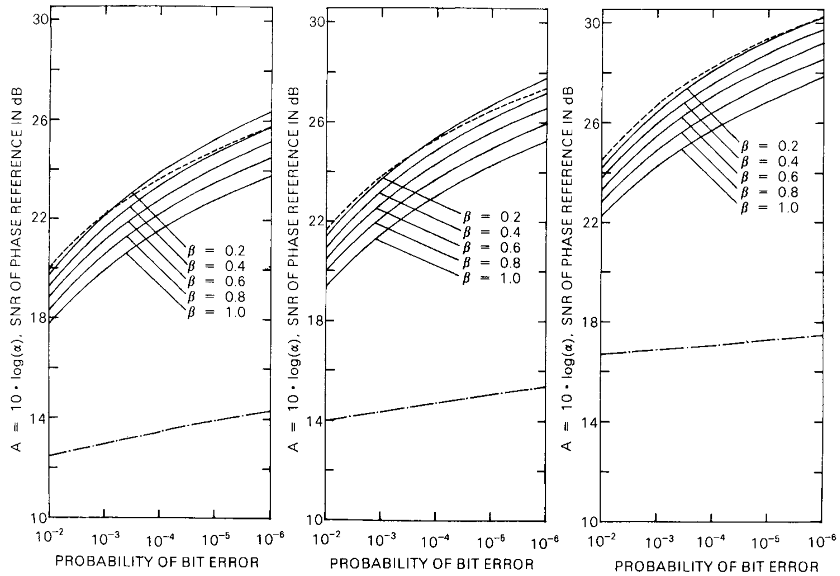

Fig. 5. Required SNR of phase reference as a function of bit error when detection loss L is specified. —— OFFSET-QPSK, ---- QPSK, —·—·— BPSK. (a) $L = 0.3$ dB. (b) $L = 0.2$ dB. (c) $L = 0.1$ dB.

The search method proposed for computing the detection loss and the required SNR of the phase reference is proved to be quite convenient.

Acknowledgment

The authors wish to thank the reviewers for their helpful suggestions in improving the earlier version of the paper.

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