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On Training Fractionally Spaced Equalizers Using Intersymbol Interpolation

FUYUN LING

Abstract—**The use of an intersymbol interpolation method in training fractionally spaced equalizers (FSE) is investigated. It is shown that the optimal interpolation filter depends on the amplitude frequency response of the transmitter filter and the channel. Using a nonoptimal interpolation filter will increase the steady-state mean-squared error (MSE) of the FSE. An interpolated complex FSE (CFSE) employing a stochastic gradient, or LMS, adaptive algorithm has very little advantage over an LMS CFSE with symbol-rate updating. However, an interpolated LMS phase-splitting FSE (PS-FSE) has a convergence speed that is twice as fast as a conventional PS-FSE. Special precautions for evaluating the performance of interpolated FSE's are discussed and a novel evaluation scheme is proposed.**

I. Introduction

The advantages of the fractionally spaced equalizer (FSE) over the symbol rate equalizer have been well recognized [1]–[3]. The main advantage of the FSE is its insensitivity to receiver sampling phase. The FSE can be implemented as a passband or baseband equalizer. In both cases, it follows a Hilbert transformer, or phase splitter, which converts the real received signal sequence into a complex sequence which is used as the input to the FSE. We call this type of FSE a complex FSE (CFSE). A variation of the CFSE, which combines the functions of both a phase splitter and an FSE into one structure, is described in [4]. We call it a phase-splitting FSE or PS–FSE.

It was proposed in [3] that it might be possible to use intersymbol interpolation to reduce the training time of an FSE. In [5], this technique was further developed for the PS–FSE by using a fast recursive least squares (FRLS) algorithm. In [6], the interpolation technique is applied to a decision-feedback equalizer for a special kind of partial-response signaling. However, the results given in [5] and [6] are only based a few computer simulations and no systematic investigation of the interpolation method has been performed.

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In this correspondence, we investigate the interpolation method for training FSE's employing the LMS algorithm, which shall be called interpolated FSE's. We first derive the optimal interpolation filter for the interpolated training. The convergence characteristics of the interpolated CFSE and PS–FSE are discussed. Problems rising from evaluation of the convergence and steady-state performance of FSE's using interpolation are considered, and a new method that correctly evaluates the performance is described. Simulation results are given to verify our analysis.

II. Intersymbol Interpolation for Training FSE's

In a baseband data communication system with a $T/2$ CFSE where T is the symbol interval, an estimate, denoted by $\hat{z}_n$, is generated every T seconds. In the training period, the z_n's are known to the receiver. The difference between z_n and $\hat{z}_n$, denoted by e_n, is used to update the coefficients of CFSE every T seconds. The input to the CFSE is the received signal sampled at every $T/2$. Thus, the signal in the delay line is shifted by two samples for each update.

The idea of intersymbol interpolation is that, if we know the desired output values of the CFSE every $T/2$ seconds, we can update the CFSE every $T/2$ seconds instead of every T seconds. More frequent updating might result in a faster initial convergence of the CFSE. In order to obtain these desired values, a noncausal interpolation filter must be used. The input to the interpolation filter is the symbols z_{n+i}, $i = 0, +1, +2, \cdots$, and its output is the desired value at nT or $nT + T/2$, denoted by $z(nT)$ and $z(nT + T/2)$. It is obvious that the output of the interpolation filter at nT, $z(nT)$ has to equal z_n. For such a filter, the folded frequency response must be a constant. Such a filter is called a Nyquist filter. However, there are an infinite number of Nyquist filters. Since the ultimate goal of the CFSE is to minimize the mean-squared error (MSE) between the symbol z_n and its estimate $\hat{z}_n$, the adaptation of coefficients at $nT + T/2$ should improve performance for the next adaption at nT. Using an arbitrarily chosen Nyquist filter as the interpolation filter may not provide such an improvement. It may even result in a larger steady-state MSE after training than a conventional FSE.

To avoid this problem, the optimal interpolation filter must also satisfy a second condition, namely, that its frequency response should be equal to the overall unaliased response of the transmitter filter, the channel, and the FSE. An arbitrary Nyquist filter may not satisfy the second condition. From [1], [3], [7], we know that the optimal $T/2$ CFSE has a frequency response that is

$$C(\omega) = F^*(\omega)/[|F(\omega-\pi)|^2 + |F(\omega)|^2 + |F(\omega+\pi)|^2 + \sigma^2] \quad (1)$$

where $F(\omega)$ is the combined baseband frequency response of the transmitter filter and the channel, and σ^2 is the variance of the noise, assuming $F(\omega) = 0$ for $|\omega| \geq 2\pi/T$, and the data symbols have a unity variance. The overall frequency response, including the CFSE, is thus equal to

$$F(\omega)C(\omega) = |F(\omega)|^2/[|F(\omega-\pi)|^2 + |F(\omega)|^2 + |F(\omega+\pi)|^2 + \sigma^2] \quad (2)$$

which is the desired frequency response of the interpolation filter.

It can be seen from (2) that the optimal interpolation filter depends on $F(\omega)$, assuming that the effect of σ^2 is negligible. In practice, the statistics of the channel are not known and the transmitter filter may or may not be known. Hence, some assumptions have to be made in choosing the interpolation filter. Degradation in the steady-state performance of the FSE will occur if the interpolation filter is nonoptimal.

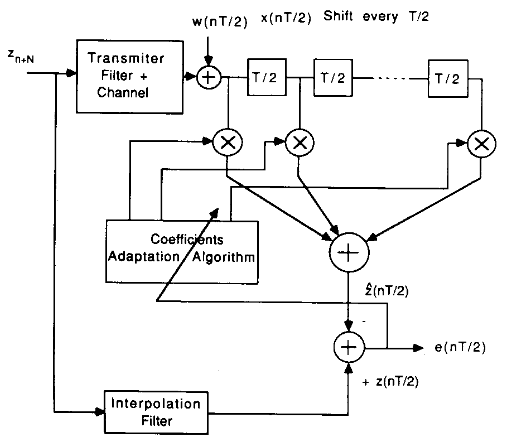

Fig. 1. Complex fractionally spaced equalizer with intersymbol interpolation.

The optimal interpolation filter for a passband CFSE can be obtained by substituting $\omega - \omega_c$ for ω in (2) where ω_c is the modulating frequency. A block diagram of a baseband model of an interpolated CFSE is depicted in Fig. 1.

As we have shown recently in [8], the optimal frequency response of a T/M PS-FSE is equal to

$$C(\omega)=F^*(\omega-\omega_c)/ \cdot \left[\sum_{-M/2\le k\le (M+1)/2} |F(\omega-\omega_c+2k\pi/M)|^2+\sigma^2\right]. \quad (3)$$

Thus, the optimal interpolation filter for a PS-FSE has the frequency response

$$C(\omega)F(\omega)=|F(\omega-\omega_c)|^2/ \left[\sum_{-M/2\le k\le (M+1)/2} |F(\omega-\omega_c+2k\pi/M)|^2+\sigma^2\right]. \quad (4)$$

III. Initial Convergence of Interpolated LMS FSE's

It has been shown [3], [7], with some approximations, that when an optimal step size is used, the excess MSE of an SRE converges according to

$$\epsilon_{ex}[(n+1)T]=(1-1/\rho N)\epsilon_{ex}(nT)+(1/\rho N)\epsilon_{opt} \quad (5)$$

where ρ is the maximum-to-average eigenvalue ratio of the autocorrelation matrix of SRE, and e_{ex} and e_{opt} are the excess and optimal MSE, respectively. N is the number of symbols that the SRE spans. Equation (5) also described the initial convergence characteristics of an LMS CFSE. However, ρ should be interpreted as the maximum-to-average eigenvalue ratio among the N nontrivial eigenvalues. The other half of the eigenvalues of a $T/2$ CFSE are equal to σ^2 which is close to zero [1], [3].

Recently, we have shown that, for a T/M PS-FSE that spans N symbols and has MN taps, $2N$ eigenvalues are not zero, and the other $(M - 2)N$ eigenvalues are approximately zero [8]. By using a slightly different method from [1], [3], we have shown that its initial convergence can be described by

$$\epsilon_{ex}[(n+1)T]=[1-1/(2\mu^2N)]\epsilon_{ex}(nT)+\epsilon_{opt}/(2\mu^2N) \quad (6)$$

where μ is the rms(root mean-square)-to-average eigenvalue ratio among the $2N$ nontrivial eigenvalues. By comparing (5) and (6) and assuming $\mu^2 \approx \rho$, we note that it would take twice as much time for a PS-FSE to converge as a CFSE.

Because the input samples to CFSE and PS-FSE are not stationary but cyclostationary [1], [3], [8], the autocorrelation matrices of data vectors at each sampling instant are not the same. Previous analysis on the initial convergence of the LMS algorithm cannot be directly applied to the convergence of interpolated LMS FSE's. While a rigorous analysis of the initial convergence of the interpolated FSE's is very difficult, we resort to evaluate their convergence performance by using computer simulation. First, we provide a heuristic discussion below.

In general, it can be shown that the optimal initial convergence of the LMS algorithm depends on the number of nontrivial eigenvalues of the data autocorrelation matrix [8]. This number is equal to the number of independent parameters to be optimized. For example, although the $T/2$ CFSE has $2N$ coefficients, its autocorrelation matrix has only N nontrivial eigenvalues, and thus it has only N independent parameters to be determined in adaptation. As a result, a $T/2$ CFSE has the same convergence speed as an SRE which has only N coefficients. An interpolated CFSE has to be optimal at every $nT/2$. Its $2N$ coefficients are then uniquely defined. Thus, there are $2N$ parameters to be optimized. On the other hand, because the autocorrelation matrices of the data vectors at $nT/2$ or $(nT + 1)/2$ each have only N nontrivial eigenvalues, the interpolated CFSE can only adjust N parameters at each update. Therefore, the interpolated CFSE, which has to optimize $2N$ parameters, will take twice as many updates as the conventional symbol-updated CFSE to reduce its MSE to the same level as the latter. Since the interpolated CFSE updates its coefficients twice as often as the conventional CFSE, their optimal convergence rates are approximately the same.

By the same argument, the optimal convergence of the PS-FSE, which has MN parameters to be optimized and is updated every T/M seconds, will be the same as the CFSE or twice as fast as the conventional PS-FSE with symbol-rate update. The convergence of the interpolated FSE's does not depend on the sampling rate of the received signal. It is shown in Section V that the above conclusions agree well with simulation results.

IV. Evaluation of the Convergence of Interpolated FSE's

The initial convergence of adaptive algorithms is usually measured by their output MSE. The output MSE is a good measure of the closeness of the coefficient vector of an adaptive filter to its statistical optimum only if the signal to be estimated is independent of the coefficients used to estimate it. This condition is satisfied for the SRE and the conventional FSE's. However, the convergence of the interpolated FSE's cannot be evaluated using the same method because the outputs of the interpolation filter are correlated. When we compute the estimate $\hat{z}_n$ of z_n, some information about z_n is already contained in the previous outputs $z(nT - mT/2)$ at $nT - mT/2$, $m > 0$ of the interpolation filter and has been used in computing the coefficients of the FSE at nT. Hence, the coefficients that were used to compute z_n fit z_n better than an unknown desired signal with the same statistics. As a result, the output MSE will not reflect how close the estimated coefficients are to their optimum. Ignoring this fact may result in misinterpretation of simulation results.

In order to correctly evaluate the convergence of interpolated FSE's during computer simulation, we introduce a duplicated FSE, which has its own input sample sequence $y(nT/2)$ and desired symbol sequence u_n. These sequences have the same statistics as the training sequences, z_n and $x(nT/2)$, for the adaptive FSE, but are statistically independent of the latter. The optimal coefficients for both FSE's are the same. The second FSE is not adaptive. Its coefficients are copied from the first FSE at each time nT during training. We shall call the adaptive and the nonadaptive FSE's the master FSE and the slave FSE, respectively. Since the desired symbol

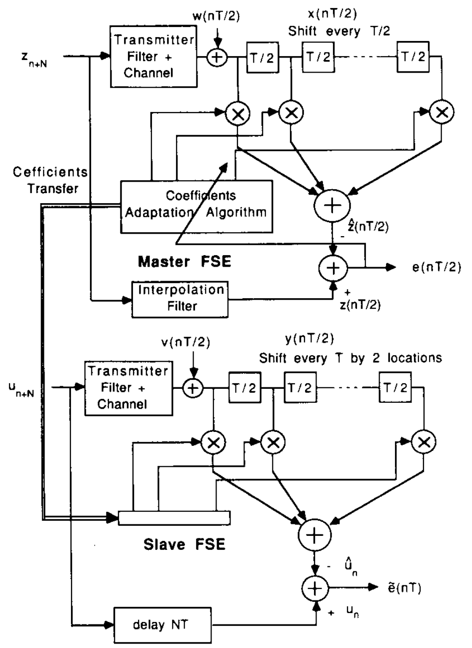

Fig. 2. Master-slave FSE scheme for evaluation interpolated FSE performance.

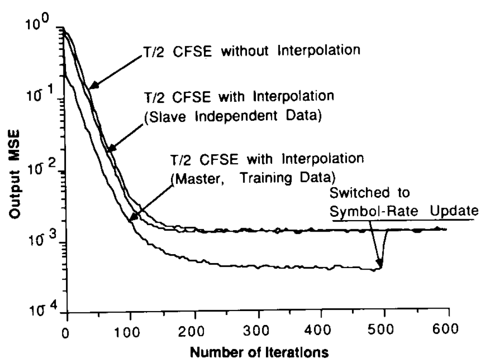

Fig. 3. Initial convergence of CFSE's.

sequence u_n of the slave FSE is independent of the training sequence z_n, the output MSE of the slave FSE is a correct measure of the convergence of the interpolated FSE coefficients. A block diagram of the scheme discussed here for a baseband model of the CFSE is given in Fig. 2. The significance of the above discussion will become clearer with the simulation results given below.

V. Simulation Results

Computer simulation has been performed to verify the analysis given above. A $T/2$ CFSE and two PS-FSE's that have sampling intervals of $T/3$ and $T/4$, respectively, were simulated. All of them span 20 symbol intervals. The overall frequency response $F(\omega)$ is chosen to be square-root raised cosine with 50 percent excess bandwidth. The optimal interpolation filter has a raised cosine spectral shape with 5-percent excess bandwidth as shown above. The signal-to-noise ratio (SNR) is 32 dB.

Fig. 3 plots the output MSE convergence curves of a $T/2$ CFSE for both an interpolated master CFSE which uses the training sequence and its slave CFSE which uses an independent testing sequence as described above. As a comparison, the output MSE of a conventional CFSE with symbol-rate

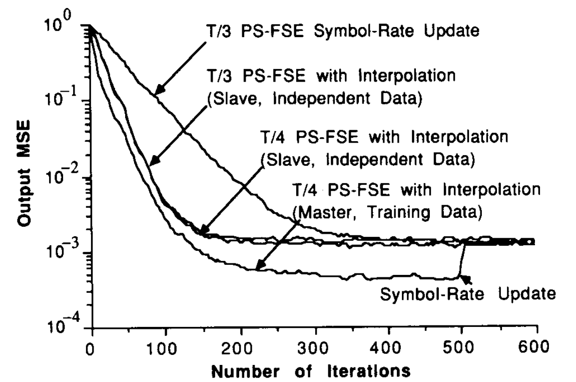

Fig. 4. Initial convergence of PS-FSE's.

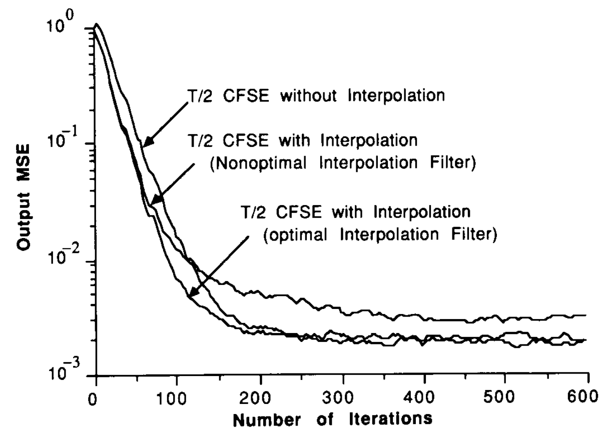

Fig. 5. Effects of optimal and nonoptimal interpolation filters.

update is also given. The rapid decline of the output MSE of the master CFSE might lead us to a wrong conclusion that the interpolated CFSE has a faster initial convergence rate and yields a smaller steady-state MSE than the conventional CFSE. Actually, this is not true, as can be seen from the output MSE of the slave CFSE. The latter correctly reflects the convergence of the interpolated CFSE and shows almost no improvement over the conventional CFSE.

As has been pointed out in [5], the noncausal interpolation method is only possible for training. When the FSE enters the data mode, it has to be switched to either symbol-rate adaptation or delayed interpolated adaptation. To show the real performance of the master CFSE in the data mode, in our simulation, starting from the 500th iteration, we switched the master CFSE to the symbol-rate update mode; its output MSE quickly increases to the same level as the slave CFSE and the conventional CFSE. Thus, the seemingly smaller output MSE of the interpolated GFSE is not a correct measure of its performance in a data mode. Interpolation with delayed adaptation will have a similar effect as symbol-rate updating. However, the adaptation step size has to be reduced to ensure stability and to yield an adequate steady-state MSE, as is discussed in [9].

Fig. 4 plots the output MSE for $T/3$ and $T/4$ PS-FSE's. It can be seen from the figure that the interpolated PS-FSE converges twice as fast as the conventional PS-FSE. Again, the convergence of the interpolated PS-FSE should be measured by the output MSE of a slave PS-FSE. The output MSE of the adative master PS-FSE cannot be directly used to measure its performance. The convergence speeds of the $T/3$ and $T/4$ interpolated PS-FSE are almost the same.

Fig. 5 shows the affects of the optimal and nonoptimal interpolation filters. The channel frequency response $F(\omega)$

used in this simulation is not square-root raised cosine; its high and low band edges have 10 and 8 dB attenuation relative to the middle of the band, respectively. For such a frequency response, the raised cosine interpolation filter is no longer optimal. In the simulation, we use the nonoptimal raised cosine filter and an optimal interpolation filter, computed according to (4). It is clear from Fig. 5 that using the nonoptimal interpolation filter yields a larger steady-state output MSE than is obtained with a conventional CFSE, while the output MSE of the interpolated CFSE using the optimal interpolation filter is similar to the latter. The convergence rates of the interpolated CFSE using the optimal and nonoptimal interpolation filters are almost the same during the first 60 symbol intervals, and are about 30 percent faster than the conventional CFSE. However, the CFSE with the nonoptimal interpolation filter slows down significantly afterwards as it approaches a higher asymptote. It is interesting to note that the convergence behavior of the interpolated FSE's is almost independent of the channel characteristics.

All the MSE curves are obtained by using average values over 100 independent runs. The step size used is computed according to $\Delta = 1/N\rho E[x^2(n)]$ or $\Delta = 1/N\mu^2 E[x^2(n)]$ where $E[x^2(n)]$ is the average power of the input samples and N is the number of total taps of the FSE's [1]–[3], [8].

VI. Concluding Remarks

An expression of the optimal interpolation filter that depends on the frequency response of the transmitter filter and the channel has been derived. We showed that using a nonoptimal interpolation filter will increase the steady-state MSE of the FSE after training, and thus degrade its performance. Since the channel frequency response is usually not known, the application of the interpolation method can be limited.

It is shown that, compared to a symbol-rate updating CFSE, an interpolated CFSE has almost no advantage for good channels and provides a slight improvement for bad channels. On the other hand, the convergence speed will increase by a factor of two for a PS–FSE using sample-rate coefficient updating and intersymbol interpolation over a conventional PS–FSE using symbol-rate updating to yield a convergence speed similar to the CFSE. However, a further increase of the sampling rate to perform more frequent updating will not accelerate the convergence speed further. The method of interpolated training would be useful if faster convergence is desired at the very beginning of training, especially for bad channels.

We have also shown that the output MSE of an interpolated FSE cannot be directly used to measure the convergence and steady-state performance of the FSE. A novel master–slave FSE's scheme for correctly evaluating the performance of the interpolated FSE is proposed.

Although only the interpolation method for FSE's employing the LMS adaptive algorithm has been discussed, some of the conclusions, such as the optimal interpolation filter and the master–slave FSE scheme, can be extended to FSE's using LS algorithms. Other aspects, such as the initial convergence of LS FSE's, both conventional and interpolated, may need more investigation.

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Generalized Multinomial Detectors for Data Communication Signals

EMAD K. AL-HUSSAINI

Abstract—**A detector with multinomial input (MN) previously derived for on-off communication systems is generalized to include binary antipodal signals with arbitrary shapes. The proposed detector is distinguished by its simpler implementation. No multiplications are needed and it has a relatively good performance. Results of numerical examples are obtained under Gaussian and non-Gaussian noise environments for different numbers of quantization levels. Solutions for *M*-ary signaling are also discussed.**

I. Introduction

The multiplications required for a digital matched filter (DMF) [1] create a problem in microprocessor implementations of medium- and high-speed voiceband modems. A number of suboptimal systems using digital techniques for signal detection [2]–[6] have been proposed that have more modest computational requirements. In [7], for an on-off radar system with constant signal amplitude, a class of detectors transforming the input sample space into a multinomial vector was considered. In this paper, it is shown how similar ideas can be applied to data transmission systems. Section II includes generalized results for antipodal signals with arbitrary shapes under Gaussian and non-Gaussian noise. Results are displayed showing comparisons to previously analyzed systems. Conclusions and suggestions for further work are discussed in Section III.

II. GMN Detector for Signals with Arbitrary Shapes

Consider the detection of two equally likely antipodal signals $+S(t)$ and $-S(t)$ over a noisy channel. The

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