Coherent Detection of *M*-ary Phase-Shift Keying in the Satellite Mobile Channel with Tone Calibration

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Abstract—**We derive a formula for the error probability of *M*-ary PSK in the satellite mobile channel when the signal is accompanied by a pilot tone which is used in the receiver for coherent detection of the signal. We compute the error probability as a function of signal-to-noise ratio, the ratio of powers in the specular and diffused signal component, the ratio of bandwidth in the pilot tone and signal extraction filters and the number of symbols, i.e., *M*, while optimizing the ratio of powers in the pilot tone and signal.**

I. Introduction

SEVERAL papers have been published recently [1]–[4] in which binary or quaternary phase-shift keying (PSK) or offset PSK (OPSK) is transmitted accompanied by a pilot tone which is used in the receiver to recover the signal in the noisy and fading channel. The channel has a Rician envelope and a random phase and contains as special cases the Rayleigh channel and the Gaussian channel. The Rician channel is used as a model for the Satellite Mobile channel, i.e., the channel between a satellite and a mobile vehicle like a car, ship or aeroplane. This paper is an extension and generalization of the previous papers in the following sense.

a) The symbols are *M*-ary.
b) We take into account the delay between the specular and diffused components of the signal.
c) There is no need for a calibration system, instead we may have hard limiters.
d) The formula for the error probability is a single integral while those of [1]–[4] are double integrals.
e) The analysis of the system is based on an adaptation of the formula of [7].

In Section II we present a model of the system and derive the essential equations. In Section III we compute the error probability for small delay. In Section IV we compute the error probability for large delay. In Section V we present a summary.

II. System Model and Analysis

A baseband equivalent (or complex envelope) model of the system is shown in Fig. 1. The input is composed of three terms: a specular component, a diffused component, and noise

$$r_i(t)=s_i(t)+d_i(t)+n_i(t). \tag{1}$$

The specular component is

$$s_i(t)=(2P_s')^{1/2}(b(t)+c^{1/2})\exp(j2\pi f_D t) \tag{2}$$

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where f_D is a Doppler frequency shift,

$$b(t)=\sum_i \exp(j\phi_i)h_s(t-iT) \tag{3}$$

is the modulated, symbol dependent signal in which $h_s(t)$ is a shaping function

$$\phi_i=a_i\pi/M \tag{4}$$

is the phase transmitted at time $iT \le t < (i+1)T$, $\{a_i\}$ is a sequence of independent, equiprobable, *M*-ary symbols from the set

$$\{a(1),\ a(2),\ \cdots\ a(M)\},\ a(m)=2m-1-M \tag{5}$$

c is the ratio of powers in pilot tone and modulated signal

$$P_s'=P_s/(1+c) \tag{6}$$

and P_s is the power in the specular component. The shaping function, $h_s(t)$ is such that the spectrum of $b(t)$ has a null at dc so that the tone and the signal can be separated in frequency. For example, if $h_s(t)$ is a biphase pulse

$$h_s(t)=\begin{cases}1 & 0\le t<T/2\\ -1 & T/2\le t<T,\quad \dfrac{1}{T}\displaystyle\int_0^T h_s^2(t)\,dt=1\\ 0 & T\le t,\ t<0\end{cases} \tag{7}$$

this assumption is satisfied, but there are many other, more sophisticated coding techniques for which this is true. The diffused component is

$$d_i(t)=\sqrt{P_d'}\,[b(t-t_d)+c^{1/2}]\xi(t) \tag{8}$$

where t_d is a delay of the diffused component relative to the specular component P_d' is related to the average power in the diffused component P_d by

$$P_d'=P_d/(1+c) \tag{9}$$

and $\xi(t)$ is a zero mean, Gaussian process with autocorrelation

$$R_\xi(\tau)=0.5\,\overline{\xi(t+\tau)\xi^*(t)},\ R_\xi(0)=1 \tag{10}$$

which represents the fading process. The upperscript * in (10) denotes the complex conjugate and the bar denotes the average value. The last term in (1) is white, Gaussian noise, with power spectral density, N_0. The noise, the fading process and the symbol dependent signal are independent random processes. The filter with impulse response $h(t)$ passes the specular and diffused component undistorted, however it bandlimits the noise. Thus, the output of the filter is

$$r(t)=s(t)+d(t)+n(t) \tag{11}$$

$$s(t)=s_i(t),\ d(t)=d_i(t) \tag{12}$$

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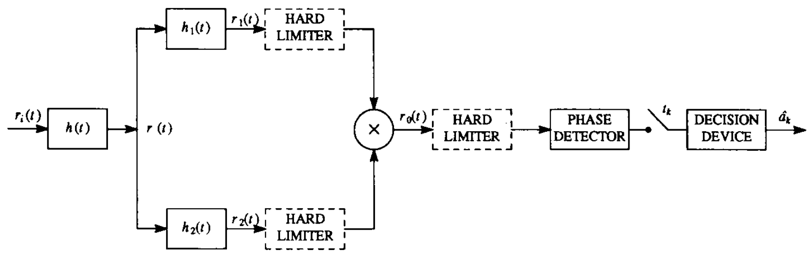

Fig. 1. Model of baseband equivalent system.

and $n(t)$ is bandlimited white noise with power spectral density, N_0.

The tone signal can be separated from $h(t)$ by passing $r(t)$ through a narrow-band low-pass filter with impulse response, $h_1(t)$, the output of which is

$$r_1(t) = r(t) * h_1(t) = c^{1/2}[(2P_s')^{1/2} \exp(j2\pi f_D t) + \sqrt{P_d'}\,\xi(t)] + n_1(t) \quad (13)$$

where * denotes convolution and

$$n_1(t) = n(t) * h_1(t) = \int_{-\infty}^{\infty} n(t-\tau_1)h_1(\tau_1)\, d\tau_1 \quad (14)$$

is narrow-band, Guassian noise, with power P_{n1} and noise bandwidth, B_{n1}.

$$P_{n1} = N_0 B_{n1}, \; B_{n1} = \int_{-\infty}^{\infty} |H_1(f)|^2\, df \quad (15)$$

where $H_1(f)$ is the Fourier transform of $h_1(t)$. The tone signal can be eliminated from $r(t)$ by passing $r(t)$ through a notch (high-pass) filter with impulse response $h_2(t)$ the output of which is

$$r_2(t) = r(t) * h_2(t) = (2P_s')^{1/2} b_2(t) \exp(j2\pi f_D t) + \sqrt{P_d'}\, b_2(t-t_d)\xi(t) + n_2(t) \quad (16)$$

where

$$b_2(t) = b(t) * h_2(t) = \sum_i \exp(j\phi_i) g(t-iT) \quad (17)$$

$$g(t) = h_s(t) * h_2(t) \quad (18)$$

and

$$n_2(t) = n(t) * h_2(t) = \int_{-\infty}^{\infty} n(t-\tau_2)h(\tau_2)\, dt_2 \quad (19)$$

has power P_{n2} and noise bandwidth, B_{n2}

$$P_{n2} = N_0 B_{n2} \quad (20)$$

$$B_{n2} = \int_{-\infty}^{\infty} |H_2(f)|^2\, df. \quad (21)$$

The only requirement on $g(t)$ is that there is a time instant t_0 such that

$$g(t_k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases} \quad (22)$$

where

$$t_k = t_0 + kT \quad (23)$$

so that at time t_k

$$b_2(t_k) = \exp(j\phi_k) \quad (24)$$

Condition (22) assures that there is no intersymbol interference (ISI) at the sampling time.

We multiply $r_2(t)$ by $r_1^*(t)$ and obtain at time $t = t_k$

$$r_0(t) = r_2(t) r_1^*(t) = c^{1/2} \exp(j\phi_k) z_2(t) z_1^*(t) \quad (25)$$

$$= c_1 \exp[j(\phi_k + \theta_2 - \theta_1] \quad (26)$$

where

$$z_i(t) = (2P_s')^{1/2} + \mu_i(t), \; i = 1, 2 \quad (27)$$

$$\mu_1(t) = [\sqrt{P_d'}\,\xi(t) + n_1(t)c^{-1/2}] \exp(-j2\pi f_D t) \quad (28)$$

$$\mu_2(t) = [\sqrt{P_d'}\, b_2(t-t_d)\xi(t) + n_2(t)] \cdot \exp(-j2\pi f_D t) \exp(-j\phi_k) \quad (29)$$

$$c_1 = c^{1/2} |z_1(t) z_2(t)| \quad (30)$$

$$\theta_i = \arg z_i(t), \; i = 1, 2. \quad (31)$$

Let

$$\psi_k = \phi_k + \Delta\theta \qquad \Delta\theta = \theta_2 - \theta_1. \quad (32)$$

The phase detector operates only on the phase to obtain the symbol a_k. Therefore, prior to multiplication in (25) or after multiplication $r_1(t)$, $r_2(t)$, and $r_0(t)$ can be hardlimited without affecting the result. The hardlimiters are therefore optional in Fig. 1.

Without any loss in generality, we may assume that $k = 0$, so that at time $t = t_0$ we make a decision about the value of a_0 from

$$\psi_0 = \phi_0 + \Delta\theta \quad (33)$$

the decision being $\hat{a}_0$. Since in the absence of noise and fading ($\mu_1(t) = \mu_2(t) = 0 \rightarrow \Delta\theta = 0$)

$$\psi_0 = \phi_0 = a_0 \pi/M = a(m)\pi/M \triangleq \phi(m) \quad (34)$$

when symbol $a(m) = 2m - 1 - M$, $m = 1, 2, \cdots M$ is transmitted, the decision rule in the presence of noise and fading) is that if

$$\phi(m) - \pi/M \leq \psi_0 < \phi(m) + \pi/M \quad (35)$$

we decide that $\hat{a}_0 = a(m)$. Thus, there is an error when $a_0 = a(m)$ is transmitted if $|\Delta\theta| \geq \pi/M$. The conditional error probability when $a_0 = a(m)$ is transmitted is thus

$$P(e|m) = P(\Delta\theta \geq \pi/M | m) \quad (36)$$

and the average error probability is

$$P_M(e) = M^{-1} \sum_{m=1}^{M} P(e|m). \quad (37)$$

In [7], was presented a formula for the distribution function of the phase difference of two Gaussian phasors. In our case, $z_1(t)$ and $z_2(t)$ are Gaussian processes therfore the formula of [7] can be applied. The result is

$$P(e|m)=F(-\pi/M|m)-F(\pi/M|m) \quad (38)$$

where

$$F(\psi|m)=-(4\pi)^{-1}\int_{-\pi/2}^{\pi/2}\exp[-WE_1(\theta)/E_2(\theta)] \cdot\left[\frac{\sin\psi}{E_1(\theta)}-\frac{|\gamma|\sin(\psi-\alpha_\gamma)}{E_2(\theta)}\right]d\theta \quad (39)$$

$$E_1(\theta)=U-V\sin\theta-\cos\psi\cos\theta \quad (40)$$

$$E_2(\theta)=1-|\gamma|\cos(\psi-\alpha_\gamma)\cos\theta \quad (41)$$

$$U=0.5(\rho_1+\rho_2)/W,\ V=0.5(\rho_1-\rho_2)/W,\ W=(\rho_1\rho_2)^{1/2} \quad (42)$$

$$\rho_i=P_s'/P\mu_i,\ i=1,2 \quad (43)$$

$$P_{\mu1}=R_{\mu1}(0)=0.5\overline{|\mu_1(t_0)|^2}=P_d'+P_{n1}/c \quad (44)$$

$$P_{\mu2}=R_{\mu2}(0)=0.5\overline{|\mu_2(t_0)|^2}=P_d'|b_2(t_0-t_d)|^2+P_{n2} \quad (45)$$

and

$$\gamma=0.5\overline{\mu_2(t)\mu_1^*(t)}/(P_{\mu1}P_{\mu2})^{1/2}=|\gamma|\exp(j\alpha_\gamma) \quad (46)$$

is the normalized crosscorrelation of the Gaussian processes $\mu_1(t)$, $\mu_2(t)$ defne in (28), (29). We can see from (39) that the error probability does not depend directly on the Doppler frequency, f_D. There is, however, an indirect dependence because the filters $h_1(t)$ and $h_2(t)$ must be able to pass the tone signal and the modulated signal, both shifted in frequency by f_D. For this reason, the bandwidth B_{n1}, must be at least $2f_{Dm}$ (f_{Dm} is the maximum Doppler frequency) which in a typical situation would be about 200 Hz [4].

It is shown in the Appendix that

$$\gamma=b_d\exp(-j\phi_0)/d,\quad \phi_0=a(m)\pi/M \quad (47)$$

where

$$b_d=b_2(t_0-t_d) \quad (48)$$

$$d=[(1+\delta/c)(|b_d|^2+\beta\delta)]^{1/2} \quad (49)$$

$$\delta=P_{n1}/P_d' \quad (50)$$

and

$$\beta=P_{n2}/P_{n1}=B_{n2}/B_{n1} \quad (51)$$

is the ratio of powers (or the ratio of noise bandwidth) of the two noises.

In these terms we obtain from (43)–(45)

$$\rho_1=K/(1+\delta/c),\ \rho_2=K/(|b_d|^2+\beta\delta),\ K=P_s'/P_d' \quad (52)$$

hence,

$$U=0.5[1+|b_d|^2+(\beta+1/c)\delta]/d \quad (53)$$

$$V=0.5[|b_d|^2-1+(\beta-1/c)\delta]/d \quad (54)$$

and

$$W=K/d. \quad (55)$$

In presenting numerical results it will be convenient to express the error probability in terms of two parameters, namely,

$$K=P_s/P_d=P_s'/P_d' \quad (56)$$

which is the ratio of powers in the specular and diffused components and

$$\mathrm{SNR}=\frac{P_s+P_d}{P_{n2}}=\frac{E/N_0}{B_{n2}T}=\frac{K+1}{P_{n2}/P_d} \quad (57)$$

which is the signal-to-noise ratio, i.e., the ratio of total signal power and noise power at the output of filter $h_2(t)$ and E is the energy per symbol. Note from (24) that if $h_2(t)$ is a matched filter to $h_s(t)$, $B_{n2}T = 1$ and SNR $= E/N_0$. The parameter K defines the nature of the channel. Thus, if $K = 0$ the channel is Rayleigh, if $K = \infty$ the channel is Gaussian and if $0 < K < \infty$ the channel is Rician. In terms of K, SNR and c (which is the ratio of powers in the tone signal and modulated signal) we obtain from (50), (9), (51), and (57)

$$\delta=P_{n1}/P_d'=P_{n1}(1+c)/P_d=\frac{1+c}{\beta}\frac{P_{n2}}{P_d}=\frac{(1+c)(1+K)}{\beta\ \mathrm{SNR}}. \quad (58)$$

Note that γ depends on symbol $a_0 = a(m)$ received at time t_0 and both γ and the other parameters depend on b_d which depends on symbol or symbols received at time $t_0 - t_d$. If $t_d = 0$

$$b_d=\exp(j\phi_0)=\exp[ja_0\pi/M] \quad (59)$$

depends also on a_0. If there is no intersymbol interference at time $t_0 - t_d$ (this will happen if $t_d = kT$ where k is a nonzero integer) so that $b_d = \exp(ja_d\pi/M)$ depends on symbol a_d only, the symbol transmitted at time $t_0 - t_d$. In the next section we shall compute the error probability when $a_d = a_0$ and in Section IV when $a_d \neq a_0$. For other values of t_d, b_d depends on many symbols in a complex way. This case will not be discussed here.

III. Error Probability when $a_d = a_0$

Since here b_d satisfies (59), (47), (49), (53), and (54) are changed to

$$\gamma=1/d \quad (60)$$

$$d=[(1+\delta/c)(1+\beta\delta)]^{1/2} \quad (61)$$

$$U=0.5[2+(\beta+1/c)\delta]/d \quad (62)$$

$$V=0.5(\beta-1/c)\delta/d \quad (63)$$

and (39) is using (55) simplified to

$$F(\psi|m)=-\frac{\sin\psi}{4\pi}\int_{-\pi/2}^{\pi/2}\exp[-(K/d)E_1(\theta)/E_2(\theta)] \cdot[E_1^{-1}(\theta)-\gamma E_2^{-1}(\theta)]\,d\theta \quad (64)$$

where $E_1(\theta)$, $E_2(\theta)$ are given in (40), (41).

In this case $F(\psi|m)$ is independent of m hence after substitution of (64) into (38) and (37) we obtain

$$P_M(e)=\frac{\sin(\pi/M)}{2\pi}\int_{-\pi/2}^{\pi/2}\exp[-(K/d)E_1(\theta)/E_2(\theta)] \cdot[E_1^{-1}(\theta)-\gamma E_2^{-1}(\theta)]\,d\theta \quad (65)$$

where

$$E_1(\theta)=U-V\sin\theta-\cos(\pi/M)\cos\theta \quad (66)$$

and

$$E_2(\theta)=1-\gamma \cos(\pi/M)\cos\theta. \tag{67}$$

Particularly, in the binary case ($M = 2$) (65) is simplified to

$$P_2(e)=\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2}\exp\left[-(K/d)(U-V\sin\theta)\right]\cdot\left[(U-V\sin\theta)^{-1}-\gamma\right]d\theta \tag{68}$$

Equation (68) should be compared to (11)–(13) of [4] where a double integral is involved.

A. Rayleigh Channel

In a Rayleigh channel $K = 0$ hence,

$$\delta=(1+c)/(\beta\ \mathrm{SNR}) \tag{69}$$

and (65), (68), are further simplified to

$$P_M(e)=\frac{\sin(\pi/M)}{2\pi}\int_{-\pi/2}^{\pi/2}\left[E_1^{-1}(\theta)-\gamma E_2^{-1}(\theta)\right]d\theta \tag{70}$$

and

$$P_2(e)=\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2}\left[(U-V\sin\theta)^{-1}-\gamma\right]d\theta$$

$$=\left[(U^2-V^2)^{-1/2}-\gamma\right]/2=(1-1/d)/2, \tag{71}$$

respectively.

B. Gaussian Channel

In a Gaussian channel $K = \infty$ hence, $\delta = \infty$, $d = \delta(\beta/c)^{1/2} = \infty$ and $\gamma = 0$ but

$$K/d=(c\beta)^{1/2}\ \mathrm{SNR}/(1+c) \tag{72}$$

$$U=0.5(c\beta+1)(c\beta)^{-1/2},\quad V=0.5(c\beta-1)(c\beta)^{-1/2} \tag{73}$$

are finite. Thus,

$$P_M(e)=\frac{\sin(\pi/M)}{2\pi}\cdot\int_{-\pi/2}^{\pi/2}\exp\left[-(c\beta)^{1/2}\ \mathrm{SNR}\ E_1(\theta)/(1+c)\right]E_1^{-1}(\theta)\ d\theta \tag{74}$$

where

$$E_1(\theta)=0.5(c\beta+1)(c\beta)^{-1/2}-0.5(c\beta-1)(c\beta)^{-1/2}\cdot\sin\theta-\cos(\pi/M)\cos\theta \tag{75}$$

and

$$P_2(e)=(c\beta)^{1/2}\cdot\int_{-\pi/2}^{\pi/2}\frac{\exp\left[-0.5\ \mathrm{SNR}\ (c\beta+1-(c\beta-1)\sin\right.}{\pi(c\beta+1-(c\beta-1)\sin\theta)}\,d\theta. \tag{76}$$

C. Numerical Results

In presenting numerical results it is convenient to express the error probability per bit

$$P_b(e)=P_M(e)/\log_2 M \tag{77}$$

as a function of signal-to-noise ratio per bit.

$$\mathrm{SNR}_b=\mathrm{SNR}/\log_2 M. \tag{78}$$

Equation (77) is certainly satisfied when we use a Gray code to encode binary data into M-ary symbols and when the resulting error probability is small. In Fig. 2 we show the optimum value of c as a function of SNR_b for various K and β. This is the portion of the average transmitted power that should be allocated to the pilot tone signal in order to obtain a minimal error probability. See also (2) and (6).

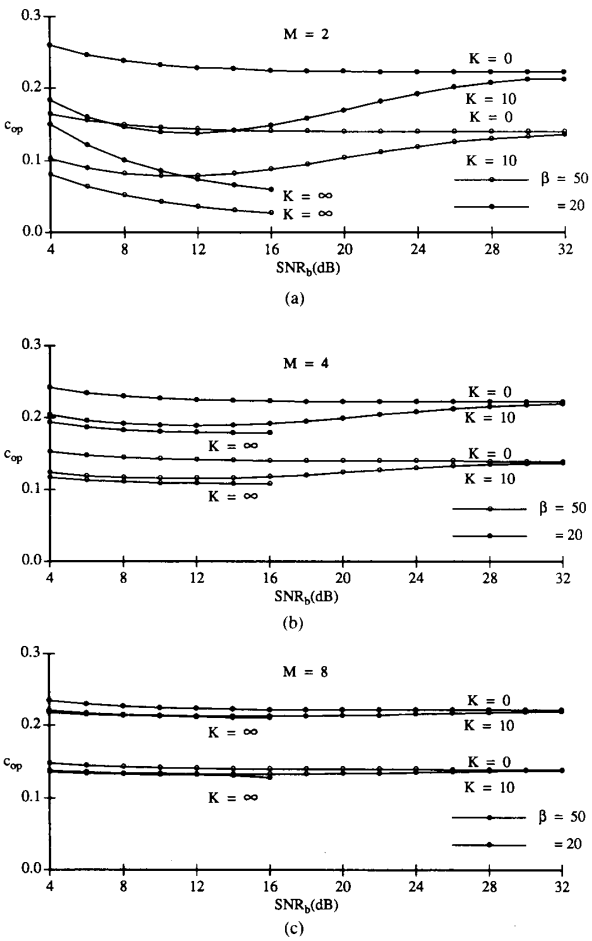

Fig. 2. Optimal ratio of powers in tone and modulated signals as a function of signal-to-noise ratio per bit for $K = 0, 10, \infty$ and $\beta = 20, 50$. (a) $M = 2$, (b) $M = 4$, (c) $M = 8$.

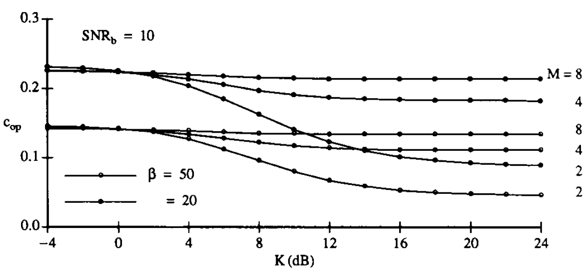

Fig. 3. Optimal ratio of powers in tone and modulated signals as a function of ratio of powers in the specular and diffused signal components for $\mathrm{SNR}_b = 10$, $\beta = 20, 50$, and $M = 2, 4, 8$.

In Fig. 3, we show the optimum value of c as a function of K. We can see from Fig. 2 that c_{op} does not change much with signal-to-noise ratio, particularly for $M = 4$ and $M = 8$, but changes significantly with β and K, the latter particularly for

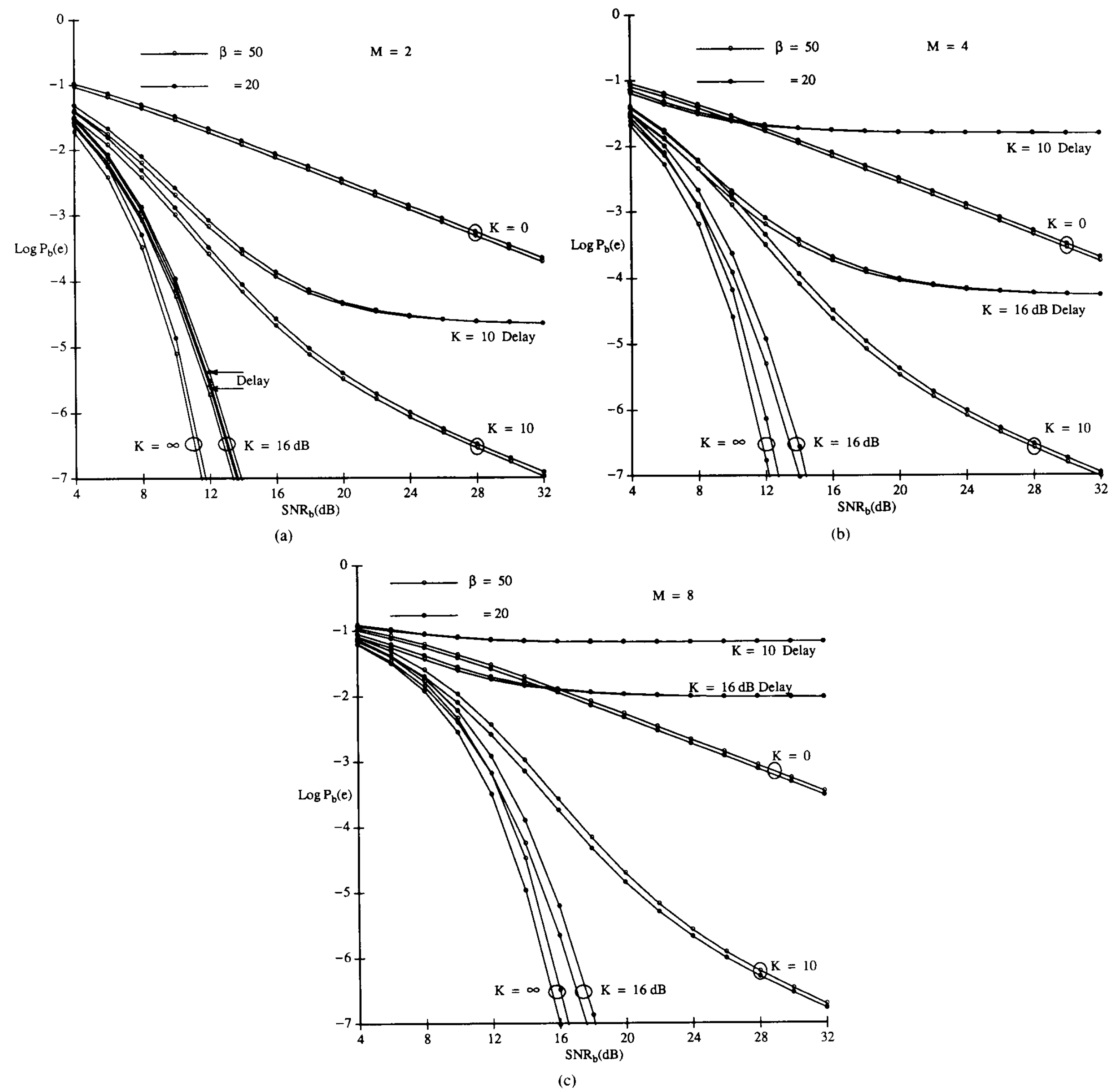

Fig. 4. The bit error probability as a function of signal-to-noise ratio per bit for $K = 0, 10, 39.8, \infty$, $\beta = 20, 50$, (a) $M = 2$, (b) $M = 4$, (c) $M = 8$. The curve labeled DELAY corresponds to the case when the symbol in the diffused component is independent of the symbol in the specular component.

$M = 2$. In Fig. 4, we show the bit error probability as a function of signal-to-noise ratio per bit for $\beta = 20, 50$, $K = 0, 10, 39.8$ (16 dB), ∞ and $M = 2, 4, 8$. The value of c was selected as shown in Table I. The curve labeled DELAY will be explained in the next section. The results are identical to those of Figs. 4-7 of [4] whenever applicable.

IV. Error Probability when $a_d \neq a_0$

Here, we assume that

$$b_d = \exp(j a_d \pi/M),\ a_d = a(l) = 2l - 1 - M,\ l - 1, \cdots M \tag{79}$$

is independent of $a_0 = a(m) = 2m - 1 - M$, $m = 1, \cdots M$. Thus, from (47)

$$\gamma = \gamma(m, l) = \exp[j2(l - m)\pi/M]/d \tag{80}$$

depends on both l and m. For each value of m there are M values of l. The conditional error probability is now

$$P(e|m) = M^{-1} \sum_{l=1}^{M} P(e|m, l) \tag{81}$$

$$P(e|m, l) = F(-\pi/M|m, l) - F(\pi/M|m, l) \tag{82}$$

TABLE I
VALUES OF c_{op} IN FIG. 4

M	β = 20				β = 50			
	K(dB)				K(dB)			
	−∞	10	16	∞	−∞	10	16	∞
2	0.23	0.15	0.10	0.09	0.15	0.09	0.05	0.05
4	0.23	0.19	0.19	0.18	0.14	0.12	0.12	0.11
8	0.22	0.22	0.22	0.21	0.14	0.14	0.14	0.14

where $F(\psi|m, l)$ is the same as in (39) with

$$|\gamma| = 1/d, \; \alpha_\gamma = 2(l-m)\pi/M. \tag{83}$$

The sets

$$\pm\pi/M^{-2}(l-m)\pi/M, \; l=1, 2 \cdots M \tag{84}$$

are the same when taken module 2π for all m. Therefore, (81) is the same for all m and we can assume $m = 1$. The average error probability is from (37)

$$P_M(e) = P(e|1)$$

$$= (M)^{-1} \sum_{l=1}^{M} [F(-\pi/M|1, l) - F(\pi/M|1, l)]$$

$$= (4\pi M)^{-1} \int_{-\pi/2}^{\pi/2} \sum_{l=1}^{M} [G_+(\theta, l) + G_-(\theta, l)] \, d\theta \tag{85}$$

where

$$G_\pm(\theta, l) = \exp[-(K/d)E_1(\theta)/E_{2\pm}(\theta, l)] \cdot \left[\frac{\sin(\pi/M)}{E_1(\theta)} - \frac{\sin((1 \pm 2(l-1))\pi/M)}{dE_{2\pm}(\theta, l)}\right] \tag{86}$$

$E_1(\theta)$ is as in (66) but

$$E_{2\pm}(\theta, l) = 1 - \cos[(1 \pm 2(l-1))\pi/M] \cos\theta/d. \tag{87}$$

Note that

$$[1-2(l-1)]\frac{\pi}{M} = [1+2(M-l+1)]\pi/M \;(\text{modulo } 2\pi) \tag{88}$$

therefore

$$\sum_{l=1}^{M} G_+(\theta, l) = \sum_{l=1}^{M} G_-(\theta, l). \tag{89}$$

In addition for each value of l, there is another value, say l' such that

$$\sin[(1+2(l'-1))\pi/M] = -\sin[(1+2(l-1))\pi/M] \tag{90}$$

while

$$\cos[(1+2(l'-1))\pi/M] = \cos[(1+2(l-1))\pi/M] \tag{91}$$

therefore (85) is simplified to

$$P_M(e) = \frac{\sin(\pi/M)}{\pi M} \int_{-\pi/2}^{\pi/2} \sum_{l=1}^{M/2} \exp[-(K/d)E_1(\theta)/E_{2+}(\theta, l)]E_1^{-1}(\theta) \, d\theta. \tag{92}$$

In the binary case, this is further simplified to

$$P_2(e) \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \cdot \exp[-(K/d)(U - V\sin\theta)](U - V\sin\theta)^{-1} \, d\theta. \tag{93}$$

The bit error probability as a function of signal-to-noise ratio per bit for $K = 10$, $M = 2, 4, 8$, and $\beta = 20, 50$ is shown in Fig. 4 as the curve labeled DELAY. The graphs for $\beta = 20$ and 50 practically coincide. For $K = 0$ and $K = \infty$ this section is irrelevant, because in the first case there is no specular component hence, there is no delay and in the second case there is no diffused component hence there is nothing to be delayed.

V. SUMMARY

We have derived a formula for error probability of M-ary PSK in the satellite mobile channel, when the modulated signal is accompanied by a pilot tone which is used in coherent reception of the fading and noisy signal. We optimized the ratio of powers in the pilot and modulated signals (c). We computed the bit error probability [$P_b(e)$] as a function of signal-to-noise ratio per bit (SNR_b), the number of symbols (M), the ratio of powers in the specular and diffused signal components (K) and the ratio of bandwidths of filters, which separate the pilot and modulated signals (β). We computed results when the diffused component is not delayed with respect to the specular component as well as for the opposite case, (i.e.) the symbol in the diffused component is independent of the symbol in the main component. We can see from Fig. 2 and Table I that when β is large c_{op} is small. For $\beta = 50$, $c_{op} < 0.15$ for useful signal-to-noise ratios while for $\beta = 20$, $c_{op} < 0.25$. For $M = 2$, c_{op}changes with K significantly as shown in Fig. 3 while for $M = 8$, c_{op} remains almost constant. We see from Fig. 4 that for $\beta = 50$ the error probability is smaller than for $\beta = 20$ when there is no delay, with delay the error probability is the same. For $M = 4$, 8 the system becomes useless when there is delay, for $M = 2$ the error probability increases significantly. When there is no delay the error probability for $M = 2$ and $M = 4$ are very close. For $M = 8$ the error probability is significantly greater only when $K = \infty$.

APPENDIX

Let

$$\gamma_1 = 0.5\overline{\mu_2(t_0)\mu_1^*(t_0)}. \tag{A.1}$$

We substitute (28) and (29) in (A-1) and obtain

$$\gamma_1 = P_d' b_2(t_0 - t_d) \exp(-j\phi_0) + 0.5\overline{n_2(t_0)n_1^*(t_0)}c^{-1/2} \exp(-j\phi_0). \tag{A-2}$$

The second term in (A-2) is using (14) and (19)

$$0.5\overline{n_2(t_0)n_1^*(t_0)}$$

$$= 0.5c^{-1/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{n(t_0 - \tau_2)n^*(t_0 - \tau_1)} h_2(\tau_2) \cdot h_1^*(\tau_1) \, d\tau_1 \, d\tau_2$$

$$= c^{-1/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_n(\tau_1 - \tau_2)h_2(\tau_2)h_1^*(\tau_1) \, d\tau_1 \, d\tau_2. \tag{A-3}$$

Since $n(t)$ is bandlimited white noise and the bandwidth of filter $h(t)$ is greater than the bandwidth of both $h_1(t)$ and

$h_2(t)$, we may write in (A-3)

$$R_n(\tau_1-\tau_2)=N_0\delta(\tau_1-\tau_2) \quad \text{(A-4)}$$

where $\delta(\ )$ is the impulse function, hence, (A-3) is using Parsevals formula (see, for example, [8] (1.2.2)) we obtain

$$\overline{0.5\,n_2(t_0)n_1^*(t_0)}=N_0\int_{-\infty}^{\infty} h_2(\tau)h_1^*(\tau)\,d\tau$$

$$=N_0\int_{-\infty}^{\infty} H_1(f)H_2^*(f)\,df=0. \quad \text{(A-5)}$$

This is zero because $H_1(f)$ and $H_2(f)$ have nonoverlapping spectra. Thus,

$$\gamma_1=P_d'b_2(t_0-t_d)\exp(-j\phi_0) \quad \text{(A-6)}$$

and the normalized cross correlation is

$$\gamma=\gamma_1/(P_{\mu1}P_{\mu2})^{1/2}$$

$$=\frac{P_d'b_2(t_0-t_d)\exp(-j\phi_0)}{[(P_d'+P_{n1}/c)(P_d'|b_2(t_0-t_d)|^2+P_{n2})]^{1/2}}. \quad \text{(A-7)}$$

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