discovered recently that Netravali and Pirsch [12] have devised an almost identical solution to this problem. The images in figure 2 include the effects of flicker suppression.

III. Summary and Conclusions

In summary, this paper has discussed the key steps required to threshold and compress graphics images digitized from standard video sources. A number of new concepts relating to shading correction, filtering and compression of the resulting image data have been discussed. While each processing step is conceptually independent, all are necessary to achieve the combination of excellent quality and high compression. With the techniques described here it is possible to transmit high quality simple graphics images over 4.8 kbit/s transmission links in about 10 s.

The concepts involved in the algorithms are not particularly complex and do not require specialized hardware for a practical implementation. They have been used in a network of videoconferencing rooms within the IBM Corporation.

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The Iterated Extended Kalman Phase Detector

SCOTT A. MERRITT

***Abstract*—Two extended range phase detectors, including the "tanlock" PD, are derived from the iterated extended Kalman filter in this correspondence. Fast converging recursion and concise initialization equations are also given. Simulations of a Wiener phase process and a first-order Markov FM process show slightly reduced mean-square error near threshold and faster phase acquisition than a sinusoidal PD.**

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I. Introduction

The application of the extended Kalman filter (EKF) to angle demodulation problems results in time-varying phase-lock loop structures [1]-[4] with sinusoidal phase detectors. Alternative, computationally intensive demodulators have been developed, such as Bucy's cyclic Bayesean estimator [5], Willsky's Fourier coefficient filter [6], Tam and Moore's smoothed EKF [7], and their Gaussian sum estimator [8]. Both [5] and [8] realize improvement relative to the PLL by propagating an approximate modulo-2π phase probability density function (pdf), not just the mean and covariance. These pdf methods calculate the conditional mean state vector by using the expected value of a nonlinear observation matrix, rather than by applying the nonlinear operation to the expected prior state vector, as is done for EKF demodulators.

The iterated extended Kalman filter (IEKF) realizes part of the improvement obtained with causal pdf methods by using the current measurement to refine the EKF linearization. The benefits of the IEKF approach are: 1) much lower computational burden than pdf methods, 2) slightly reduced mean-square error compared to a sinusoidal PD, 3) faster phase acquisition than a sinusoidal PD, 4) the extended range PD of [9]-[11] and the "tanlock" PD can be derived from the IEKF PD, 5) it offers an explanation of the improvement in performance obtained with "derivative control" as requested in [11], and 6) it can be incorporated into computationally intensive smoothing estimators.

II. Iteration Equations

A. Formulation

Consider a discrete-time system with linear dynamics and a nonlinear observation matrix given by

$$\boldsymbol{h}_k(\boldsymbol{x}_k) = a_k \begin{vmatrix} \cos(\theta_k) \\ \sin(\theta_k) \end{vmatrix} \tag{1}$$

where a_k and θ_k are components of the state vector $\boldsymbol{x}$ and k is the discrete-time index [1]-[2]. Rather than compute the expected value of $\boldsymbol{h}_k$ to obtain a minimum variance estimate of $\boldsymbol{x}_k$ [12], the IEKF algorithm uses the current (kth) measurement to relinearize the observation matrix around $\boldsymbol{x}_{k,i}$ starting with $\boldsymbol{x}_k(-)$. The IEKF equations [12] simplify considerably if the signal amplitude is constant or slowly varying since then only the phase state update is required to relinearize the observation matrix. All other state variables are computed from the converged phase update. The iterated state update equation therefore reduces to the following IEKF PD recursion:

$$\hat{\rho}_{i+1} = \frac{D|y|}{a} \sin(\psi' - \hat{\rho}_i) + D\hat{\rho}_i \tag{2}$$

where the dependence on k has been supressed for conciseness, i is the iteration index, $|y|$ is the magnitude of the received noisy sample $|y| \exp(j\psi)$, ψ' is the phase lead of the kth sample with respect to the "L.O." phase $\hat{\theta}(-)$, i.e. $\psi' = (\psi - \hat{\theta}(-)) \bmod 2\pi$, $\hat{\rho}_i = (\hat{\theta}_i - \hat{\theta}(-)) \bmod 2\pi$ is the PD output, and D is a demodulator gain factor given by $D = a^2 P(-)/(a^2 P(-) + r)$ where $P(-)$ is the prior phase error covariance and r is the noise covariance.

We shall now examine an approximate solution of (2) in order to understand the asymptotic behavior of the new PD. If the PD output is small, then we may approximate $\cos(\hat{\rho}_i)$ by one and $\sin(\hat{\rho}_i)$ by $\hat{\rho}_i$. Taking the Z transform of the approximated equation and applying the final value theorem, we obtain the "tanlock" PD characteristic [11]

$$\hat{\rho}_\infty(|y|, \psi') = \frac{\frac{D}{a(1-D)}|y| \sin(\psi')}{1+\frac{D}{a(1-D)}|y|\cos(\psi')}. \quad (3)$$

Since the assumption used to obtain (3) is rather restrictive, let us examine what happens when (2) has nearly converged. When $\hat{\rho}_{i+1} \approx \hat{\rho}_i$ we may rearrange terms to obtain the relation postulated by Acampora and Newton [9]

$$\hat{\rho}(|y|, \psi') = \gamma(|y|) \sin(\psi' - \hat{\rho}(|y|, \psi')) \quad (4)$$

where $\gamma(|y|) = D|y|/a(1-D)$ is called the iteration gain. A fast converging form of (4) can be obtained from the Newton-Raphson method, resulting in the following recursion:

$$\hat{\rho}_{i+1} = \gamma \frac{\hat{\rho}_i \cos(\psi' - \hat{\rho}_i) + \sin(\psi' - \hat{\rho}_i)}{1+\gamma\cos(\psi' - \hat{\rho}_i)} \quad (5)$$

where some functional dependencies have been suppressed for conciseness.

B. Initialization and Solution of the IEKF Recursions

A suitable initialization function may be obtained by upper bounding $\sin(|\psi' - \hat{\rho}|)$ with a parabola, and substituting the bound into (4). The solution to the resulting quadratic equation is

$$\hat{\rho}_0 = \left\{ |\psi'| - \frac{\pi}{2} + \frac{\sqrt{\pi^4 - 16\gamma\pi^2\left(|\psi'| - \frac{\pi}{2} - \gamma\right)} - \pi^2}{8\gamma} \right\} \cdot \operatorname{sgn}(\psi'). \quad (6)$$

The solution obtained for the fast IEKF recursion (5) has some very interesting properties (Fig. 1). First, the IEKF PD approaches an arctangent for large values of the iteration gain, γ, and approaches a sinusoid for small values. Since γ is inversely proportional to the noise covariance, one should therefore expect similar performance from the IEKF and EKF at low CNR's. Similar performance should also be expected at high CNR's since the IEKF and EKF phase errors are small and the sinusoidal and arctangent characteristics may both be replaced by the small angle approximation. Second, a discontinuity appears at $\psi' = \pi$ for "large" values of the received signal amplitude, i.e., when $\gamma > 1$. This discontinuity is due to the multiple-valued nature of the solution to (4) or (5) for these conditions (Fig. 2).

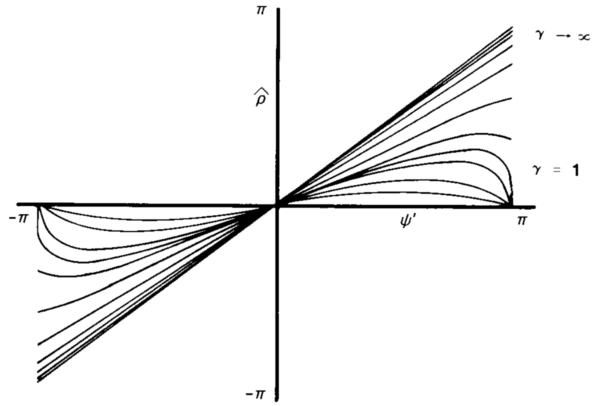

Fig. 1. Iterated extended Kalman phase detector characteristic.

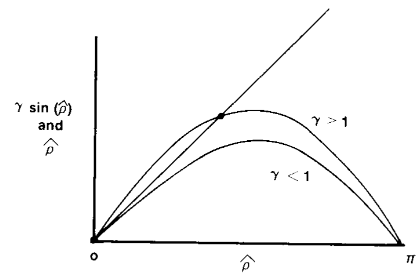

Fig. 2. Two solutions of $\hat{\rho} = \gamma \sin(\hat{\rho})$ in the range $[0, \pi)$ for $\gamma > 1$ and $\psi' = \pi$.

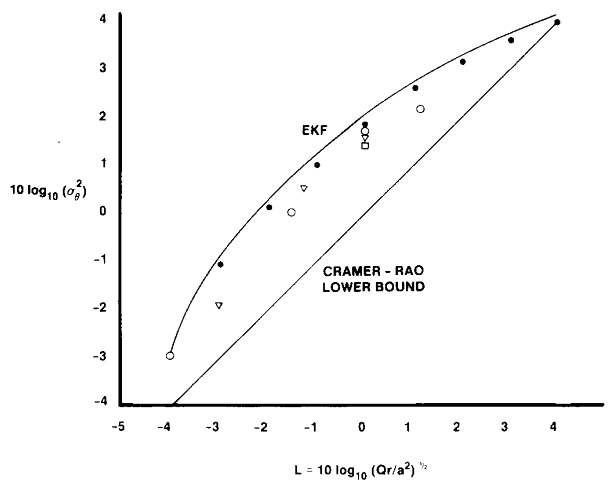

Fig. 3. Phase MSE for Wiener phase process ($a^2 Q/r = 0.01$) ● IEKF; ▽ Bucy; ○ Willsky; □ Tam and Moore ($M = 6$).

III. Simulation Results

A. Mean-Square Error

The mean-square error performance of the EKF and IEKF was evaluated by simulation of a Wiener phase process as in [5]-[7], [13], [14] and a first-order Markov frequency modulated signal as in [1], [2] and [4]. The signal amplitude a_k was fixed in both cases.

The results for the Wiener phase process were obtained by taking 10 samples per time constant for the corresponding analog PLL, i.e., $a^2Q/r = 0.01$ where Q is the incremental phase variance [14] times the sampling period and r is the noise covariance. Refer to [4], [5], and [13] for further sample rate considerations. The demodulator performance metric is the mean-square modulo-2π phase error σ_θ^2 computed as a function of the "noise-to-signal" ratio, $L = 10*\log(Qr/a^2)^{1/2}$ (Fig. 3). The results of Bucy [5], Willsky [6], and Tam and Moore [7] are also given for comparison. The improvement of the IEKF PD over the EKF's sinusoidal PD, 0.15 ± 0.06 dB at $L = 0$, represents a fraction of the improvement obtained by the causal pdf-based estimators (about 0.60 ± 0.15 dB) but the IEKF improvement is possible at much lower computational cost. Additional preliminary results indicate that the IEKF is more robust with respect to Q than the EKF.

The IEKF and EKF performance in the first-order Markov problem ([1], [2], and [4]) was determined as a function of CNR in the message's equivalent noise bandwidth for modulation indexes of 25 and 100. The performance metric was inverse mean-square FM message error E_λ^{-1} as in [1], [2], and [4]. (Note that E_λ^{-1} does not increase 1 dB/dB above threshold

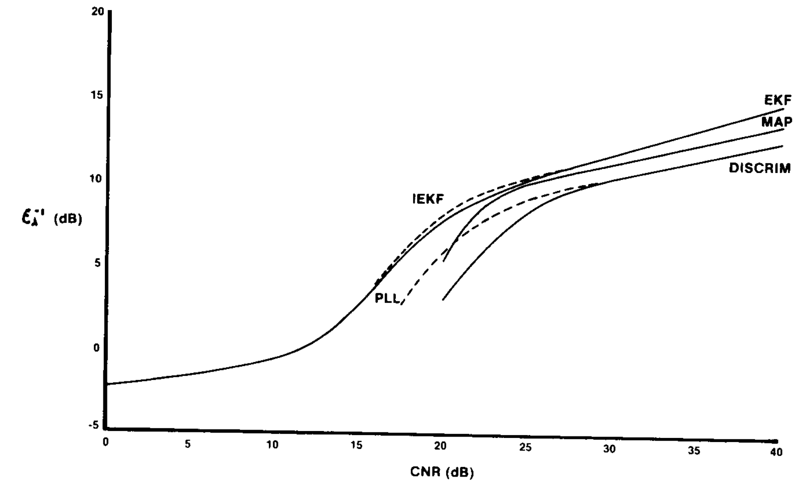

Fig. 4. E_λ^{-1} versus CNR, $\beta = 25$.

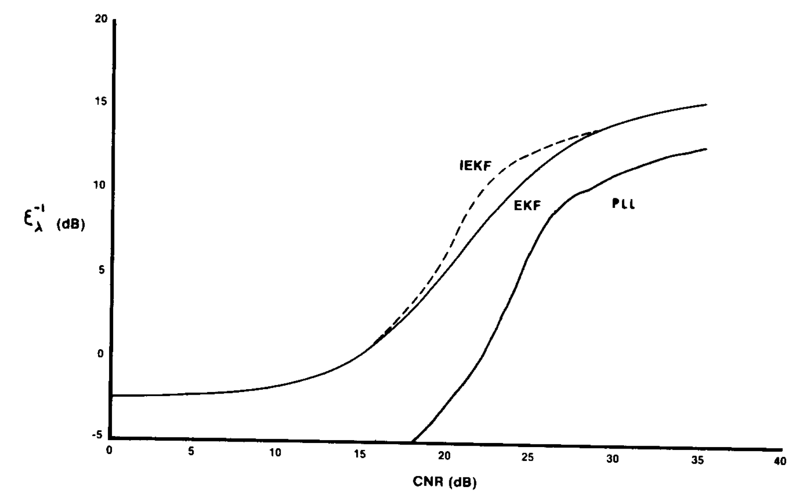

Fig. 5. E_λ^{-1} versus CNR, $\beta = 100$.

for this message spectrum [15].) Figs. 4 and 5 show that the EKF and IEKF outperform a discriminator, a DPLL optimized according to the Jaffe–Rechtin criterion and the recursive MAP demodulator of [1]. Most of the IEKF PD improvement occurred near FM threshold, as expected, and resulted in a threshold extension of approximately 2 dB relative to the sinusoidal PD when the modulation index was 100. This is an example of the phase detector linearity required to achieve the FM improvement factor for large modulation indexes.

An EKF fixed-lag smoother was also implemented to verify it's performance in our FM demodulation problem. As in [7], the fixed-lag smoother exhibited substantial improvement in mean-square FM message error for adequate lag values but the improvement was mainly restricted to suprathreshold CNR's. Smoothing also incurred a substantial computational cost. However, if a smoother is desired, additional improvement should be possible by using the IEKF PD instead of the EKF's sinusoidal PD to extend the threshold of the zero-lag (causal) estimator incorporated in the smoother.

B. Acquisition

The phase acquisition behavior of the IEKF and EKF was studied for the case of a Wiener phase process using an initial phase estimate uniformly distributed over $[-\pi, \pi)$. Fig. 6 shows the rms phase error trajectory which was obtained by averaging 300 acquisition sequences at $L = -12$ dB. The IEKF PD exhibited improved acquisition time with no loss in steady-state mean-square error performance. The difference in these curves is not large because the two PD characteristics are distinct for only a portion of the acquisition sequence. In other words, the IEKF PD initially had an arctangent characteristic (i.e., the average iteration gain was larger than one) then the PD reverted to a sinusoidal characteristic as the demodulator gain converged to it's steady-state value.

C. Convergence Results

Using (6) for the initialization function, the fast IEKF recursion (5) converged to within 10^{-5} radians in only two or three iterations (Fig. 7). Recursion (4) requires more iterations than (5) and is more dependent on L.

IV. Conclusions

A new phase detector characteristic (2) results when the iterated extended Kalman filter algorithm is used to design coherent demodulators. The extended range phase detector of Acampora and Newton and the "tanlock" phase detector can be derived from the iterated phase detector. The improvement in acquisition and steady-state mean-square error performance is modest but this technique incurs much less computational cost than pdf or smoothing methods.

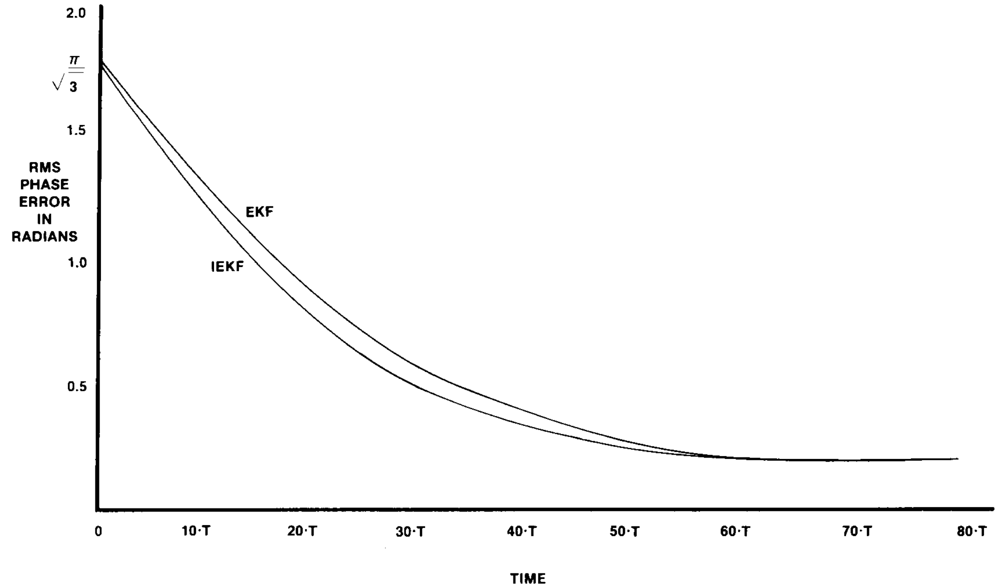

Fig. 6. Phase acquisition for EKF and IEKF, random initial phase, $L = -12$ dB.

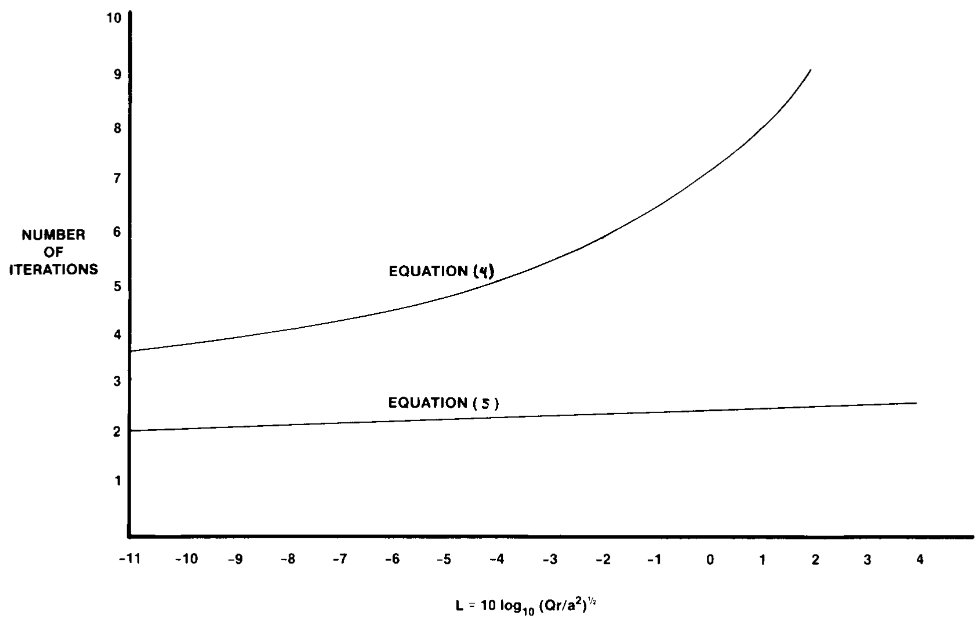

Fig. 7. Number of iterations required for convergence to $\pm 10^{-5}$ radians.

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A Limited Sensing Random-Access Algorithm with Binary Success-Failure Feedback

MICHAEL PATERAKIS AND P. PAPANTONI-KAZAKOS

Abstract—**We consider the problem of random access communication over a time slotted channel, with binary success/failure feedback. The feedback informs the users only whether or not there was a success (single transmission) in the previous slot.**

For the above problem we propose and analyze a limited feedback sensing algorithm (each user is required to observe the channel feedback, from the time he generates a packet to the time that this packet is successfully transmitted). The algorithm requires central control implemented by a central receiver.

The limit Poisson user model is adopted. The algorithm achieves a throughput of 0.322 and induces low delays for relatively low input rates.

I. Introduction

We consider the case where a large number of bursty, independent, possibly mobile, packet transmitting users wish to communicate with each other or with a central receiver, by transmitting messages over a common channel (e.g., a satellite or packet radio link, a coaxial cable, etc.). We assume that the channel is slotted, and that the users are synchronized so that transmissions can start only at the beginning of a slot.

By the end of each slot, a binary feedback informs all the active users (users with a packet for transmission) whether or not that slot contained a single packet transmission. This is known as success/failure (SF) feedback. This type of feedback arises when the users attempt to disguise the fact that they are transmitting by keeping the transmitted power very low. In this case, a collision of two or more transmitted signals results in a noise-like waveform that is difficult to distinguish reliably from pure channel noise. (See [5], [6].)

The solution to the above problem must incorporate a distributed scheme, termed random-access algorithm, (RAA), for allocating the channel bandwidth among the users. The key performance measures of a RAA are its throughput and delay characteristics.

The SF binary feedback is the poorest kind of binary feedback, as compared to the collision/noncollision (informs the users whether or not there was a collision in the previous slot) and the something/nothing (informs the users whether or not the previous slot was empty) feedbacks. The main problem with SF feedback is asynchronicity in channel history information among the active users. When a failure feedback is received, the users who transmitted (if any) in the corresponding slot recognize it as a collision, while those who did not transmit cannot discriminate between collision and emptiness. This fact has been early recognized by the researchers in the field. In [5], it is claimed that it is not possible to design a stable RAA for a system with SF feedback, unless some external control mechanism is deployed. In [6], a finite number of users is considered, and an algorithm based on generalized group testing methodologies is proposed. The algorithm operates cyclically and the collisions (if any) between packets that arrive during one cycle are resolved during the next cycle. At the beginning of a cycle, each user may have a packet for transmission with some fixed probability p, which is independent of the length of the previous cycle. Furthermore, an "auxiliary" user is used in the operation of the algorithm. The auxiliary user transmits a fictitious packet in the first slot of every cycle.

To the authors' knowledge, none of the existing dynamic control policies used to stabilize the slotted ALOHA protocol [1], apply for S/F feedback. (See [4, Sect. B.2].) Furthermore, polynomial and exponential backoff schemes for the slotted ALOHA protocol have been proven unstable for the limit Poisson user model ([14], [15]). Of course one can consider the finite population user model. In [2], it was proven that the ALOHA system with M identical and independent Poisson users is stable if the retransmission probability is $1/M$. The algorithm then attains throughput equal to $(1-1/M)^{M-1}$. To achieve stable operation, each user must know the total number of users in the system. Furthermore, the delays of this scheme increase monotonically with M, even if the total input rate is very small, [3]. Finally, in [16] it is shown that any superlinear polynomial backoff protocol (e.g., quadratic backoff), is stable as long as the cumulative arrival rate is less than one and the number of stations is finite. The delays, however, increase superlinearly with the number of users in the system. In contrast, the performance of RAA's designed under the limit Poisson user model is independent of the number of users, and it is not required that each user know the user population. This is especially important in a system where the users are mobile, and can thus enter and leave the system freely. In addition, the infinite user population assumption provides us with an upper bound to the delay that can be achieved with a finite number of users, [17]. Furthermore, in [9] it was shown that, for a large class of RAA's as the population size increases, the stability of an algorithm in the class is determined by the throughput of the algorithm under the limit Poisson user model.

In this paper, a stable limited sensing RAA for the limit Poisson user model is proposed and analyzed. The algorithm works with binary SF feedback. The algorithm requires central control implemented by a central receiver, which transmits a fictitious packet after each F feedback slot. This converts SF feedback to E-S-C (empty versus success versus collision) feedback. The organization of the paper is as follows: In Section II, we present the system model. In Section III, we describe the operation of the algorithm. In Section IV, the results of the throughput and the mean delay analyses are presented. Section V contains the conclusions.

II. System Model

Transmissions by the users are assumed synchronous; that is, they can only start at the beginning of a slot. When a single

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